Spatial Chaining Methods for International Comparisons of Prices and Real Expenditures

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PPPs from ICP 2011

Country		PPP	PLI% (World=100)
P.R. China		3.506	70.0
Hong Kong		5.462	90.5
India		15.109	41.7
Australia		1.511	201
Japan		107.454	173.6
Luxembourg		0.906	162.4
Ethiopia		4.919	

Real and Nominal per capita GDP
(in US dollars)

Country		Real GDP 2011	Nominal GDP 2005	Nominal GDP 2011
P.R. China Hong Kong India				

Objectives- continued

- To improve upon the method of minimum spanning trees for determining the links
    - Spanning trees are generally unstable
    - Links obtained are not necessarily intuitive
    - The Hill method does not necessarily give the best possible binary comparisons
- In this paper we introduce the notion of shortest path comparisons between pairs of countries
    - Implement the new concept using different measures of reliability
    - Examine the differences in the links between MST and Shortest path (SP) methods
- We establish a link between weighted GEKS and MST and SP methods of linking
    - We establish algebraic equivalence between MST comparisons and weighted GEKS

Weighted GEKS

- GEKS is based on the premise that a direct binary comparison is the best way to compare two countries.
- GEKS provides transitive comparisons that are the closest to the binary comparisons
- Given that ICP covers the whole world – comparisons are

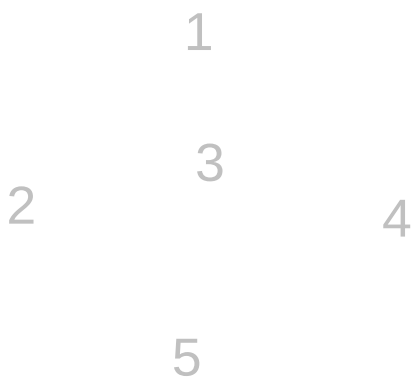
Spatial Chaining

- For temporal comparisons, we have a natural order to chain comparisons

  2010 2011 2012 2013 2014 2015
- Spatial chaining is where countries or regions are compared with other countries using chained links
    - In spatial comparisons, there is no natural ordering
- How does one order the countries to determine the chains?
- Question then is whether it is possible to devise a method of finding spatial chains to making comparisons between countries.
- Hill (1999, 2001, 2004, 2009) advocated the use of spatial linking based on minimum spanning trees.
    - Spanning tree is a concept used in Graph Theory
    - Spanning tree provides an order which countries can be linked.

Price comparisons using a Spanning tree

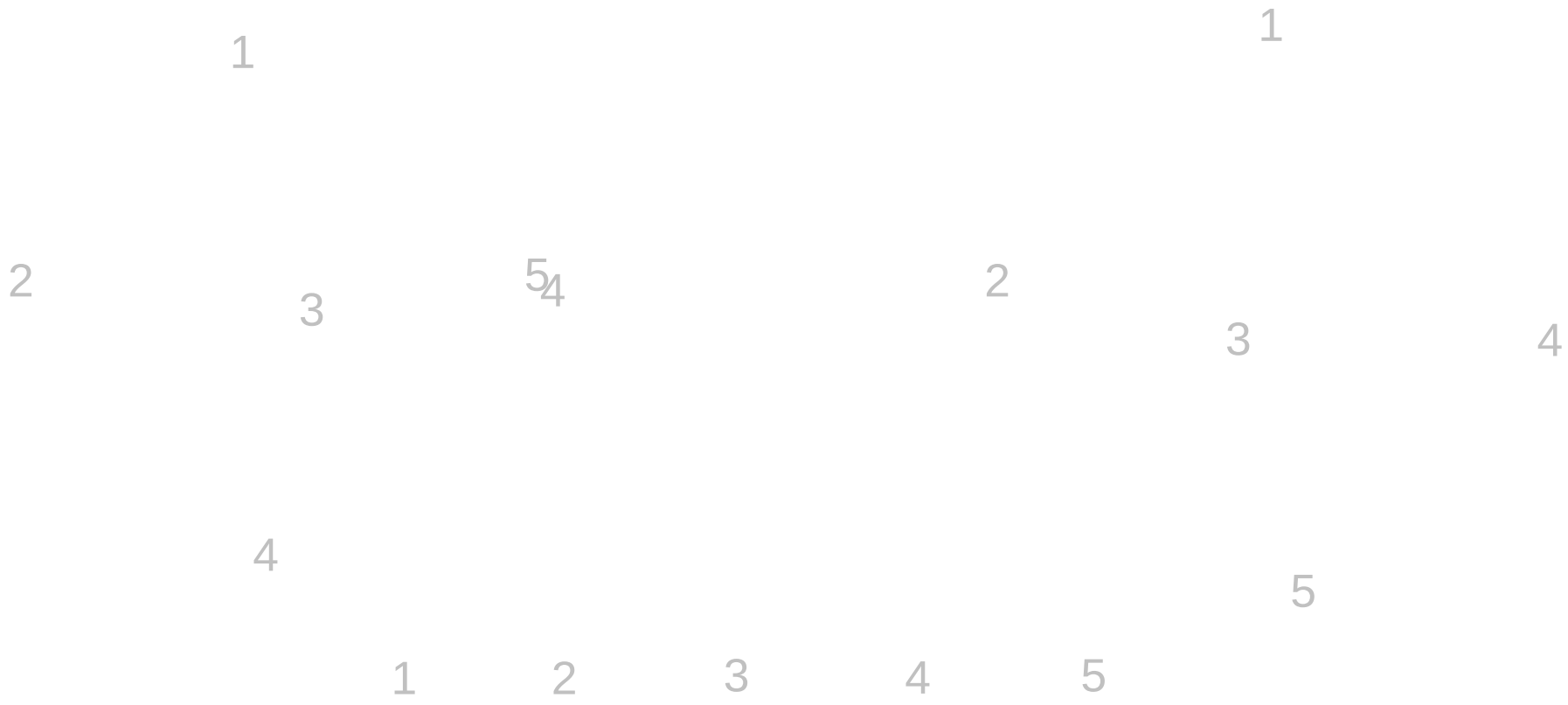
- First we choose a binary index number that satisfies time/country reversal test– e.g., Fisher and Tornqvist.
- A spanning tree is a connected graph where there is an unique path between any pair of countries.
- Suppose we wish to use the following spanning tree for a set five countries.



- The comparisons between countries are made using the chains shown in the spanning tree.

$$P_{12}^{ST}(F) \quad P_{14}^{F} \quad P_{43}^{F} \quad P_{32}^{F} \qquad P_{15}^{ST}(F) \quad P_{14}^{F} \quad P_{45}^{F} \qquad P_{35}^{ST}(F) \quad P_{34}^{F} \quad P_{45}^{F}$$

Price comparisons using a Spanning tree



Weighted GEKS and MST Price comparisons

We prove the following two theorems:

Theorem 1: Consider a spanning tree that connects all the countries. Let w_{jk} represent weights such that $w_{jk} = 1$ if country j is directly connected to country k and zero otherwise. Then price comparisons based on the MST are identical to the indexes obtained using weighted GEKS method with weights w_{jk} implied by the MST- can be proved using induction.

Theorem 2: Consider the following system of generalized weights

In the limit as x tends to infinity, the weighted-GEKS method converges to the minimum spanning tree method

Spatial chaining and CPD

- When it comes to spatial chaining the following question is often raised:

  Is it meaningful to obtain spatially chained comparisons between

Spatial chaining and CPD

We consider the following scenario:

Theorem: The PPPs computed for this data matrix using CPD method and spatial chaining are identical.

Proof uses the structure of data and the algebraic derivation of PPPs using the CPD method

Which spanning trees to use?

- For a given set of M countries, there can be M^{M-2} number of spanning trees that can be used. For example, if there are five countries, there can be 125 different spanning trees.
- Which spanning tree should we choose?
  - Hill (1999) and subsequent work advocates the use of minimum spanning tree (MST) for price comparisons.
- To identify the minimum spanning tree, we need to associate weights to each binary comparison. This is like a measure of cost associated with the comparison.
- In rest of this work, we make use of the three distance measures described before – LPS; WPRD and Allen-Diewert measures.
  - The minimum spanning tree is identified using Kruskal's algorithm.

Minimum Spanning tree - example

Shortest Path (SP) Approach

- Main starting point is that MST may actually make some comparison worse than the original binaries.
- The shortest path between a pair of countries i and k is

Which distance metric do we choose?

-

Shortest Path (SP) Approach

- If the MD path between two countries j and k is defined by countries with labels $\{i_1, i_2, \ldots, i_P\}$, then

- Properties:

1.

2.

3. is a proper distance metric

4. The SP chained index is not transitive – by construction. So we can use GEKS on the SP index.

5. The Shortest Paths.003 Tc5] Twd . det4Td [(i)1.8(f)-1.3(5(.6(ne)1.9(d

Empirical Results

Data used:ICP 2011 data for Household

Empirical Results

We construct the following set of comparisons:

- MST (LPS)
- MST (WRPD)
- Shortestpath GEKS (LPS with L>P)
- Shortestpath GEKS (WRPD)
- Weighted GEKS (with weights of 1/(1+LPS))
- Weighted GEKS (with weights of 1/(1+WRPD))
- Weighted GEKS (on matrix of ones and zeros derived from union of SPSTs– LPS with L>P)
- Weighted GEKS (on matrix of ones and zeros derived from union of SPSTs– WRPD)

MST with LPS distance measure

MST with weighted relative price distancemeasure

The MD Paths from Selected Countries Using LPS Measure

Morocco with all other countries

The MD Paths from Selected Countries Using LPS Measure

Kazhakistan with all other countries

Union of all Minimum Distance Paths -WPRD

Comparisons with LPS

	Total within region comparisons	Shortest path without external countries	MST without external countries
Africa	1225	83	31
Asia_Pacific	253	17	7
CIS	36	11	5
EU_OECD	1035	57	22
Latin America	120	24	6
West Asia	55	3	2
Singleton	0	0	0

	Total within region comparisons	Shortest path without external countries	MST without external countries

Comparisons with WPRD

Country PPP WRPD MD

Robustness of comparisons
Various methods

We use JackKnife method to assess stability of comparisons from various methods. Results are reported below

	PPP LPS SP	PPP LPS SP GEKS	PPP LPS MST	PPP LPS MST WGEKS	PPP LPS SP WGEKS
CHN	0.2730	0.1650	0.7836	0.7836	0.0820
FJI	0.1898	0.0455	0.2655	0.2655	0.0278
HKG	0.2536	0.1968	1.5271	1.5271	0.1711
IND	2.0817	0.9498	5.7680	5.7680	0.4384
IDN	757.2862	184.8605	919.6091	919.6091	243.8574
LAO	281.2256	151.5773	865.4435	865.4435	89.4408
MAC	0.2417	0.1962	1.1435	1.1435	0.2782
MYS	0.0658	0.0470	0.3034	0.3034	0.0288

We are currently conducting simulation studies to assess the performance of various methods in the presence of noise in price data.

Conclusions

Spatial chaining is shown to be a promising area for research.

- The SP approach provides better links between pairs of countries than the MST.
- The SP links are more stable than the MST links.
- We are able to provide a link between spanning tree comparisons and weighted GEKS methods.
- Of all the distance and similarity measures we find LPS and WPRD to be conceptually suitable for the SP approach.
- We are currently conducting a simulation study to assess the robustness of the SP comparisons in the presence of noise in the price and expenditure observations.
- Given the stability of shortest path chains between countries, it

Thank you!