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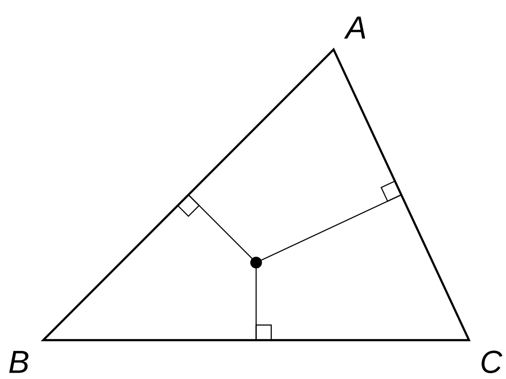
MATHEMATICS ENRICHMENT CLUB.
Problem Sheet 5, June 4, 2018

1. If a and b are positive integers with $a > b$, and $(a+b)^2 - (a-b)^2 > 29$, find the smallest possible value of a.

2. If the straight line $y = x + c$ meets the circle $x^2 + y^2 = 1$ at a single point, find the value(s) of c.

3. Let ABC be a triangle. Prove that the perpendicular bisectors of the sides AB, AC and BC intersect at a single point. (This point is called the circumcentre of the triangle.)



4. Without using a calculator, show that
$$\sqrt[3]{5\sqrt{13}+18} - \sqrt[3]{5\sqrt{13}-18} = 3.$$
Hint: Let $x = a - b$ and then cube.

5. If x and y are positive integers which satisfy $x^2 - 8x - 1001y^2 = 0$, what is the smallest possible value of $x + y$?
(AMC 2012 Senior Division Q23)

Senior Questions

1. Suppose that $g(x)$ is an odd function. Show that, if g is de ned at $x = 0$, then $g(0) = 0$.

2. (a) Suppose that $f(x)$ is an even function de ned for all real x and di erentiable throughout its domain. Show that $f'(x)$ is an odd function.

   (b) Similarly, suppose that $g(x)$ is an odd function de ned for all real x and di eren-tiable throughout its domain. Show x
   (x) is an odd function.