MATHEMATICS ENRICHMENT CLUB.
Problem Sheet 9, July 24, 2017

1. What is the least positive integer n such that 90 n is a cube?

2. Show that any straight line passing through the centre of a parallelogram (i.e. the intersection of the diagonals) divides the parallelogram into two equal areas.

3. A mathematics test has 5 questions on each of which people can score 0,1,2 or 3 marks. How many ways can a student receive a total of 12 marks for the test?

4. Use the fact that $2xy = (x + y)^2$ x^2 y^2 to show that

$$2(b \quad c)(c \quad a) + 2(c \quad a)(a \quad b) + 2(a \quad b)(b \quad c) \quad 0$$

for all real numbers $a; b; c$.

5. Take any triangle ABC and show how to construct an equilateral triangle inside ABC whose vertices touch the sides of ABC. (Hint: Start by constructing an equilateral triangle outside ABC with AB as one of its sides.)

6. Imagine that we have a nite set A of integer numbers, that is, a collection of integers without repetition. Consider the set $A + A$ of all possible sums of two numbers in A:

$$A + A = fn : \ n = a + a' \text{ for some numbers } a; a' \text{ in } Ag:$$

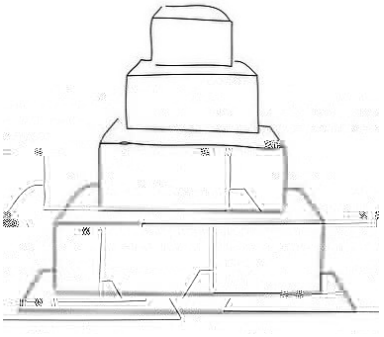
We denote how many numbers there are in the set A by jAj.

(a) Show that $jA + Aj$ $2jAj$ 1.

(b) Show that if $jA + Aj = 2jAj$ 1, then A is an arithmetic progression.

Senior Questions

1. Imagine that you have a square based cake, like the one in the picture.



   (a) How would you cut it into 5 pieces of equal volume? How about 7 pieces?

   (b) How about n pieces of equal volume?

2. Show that $\log_2 3$ is not a rational number.