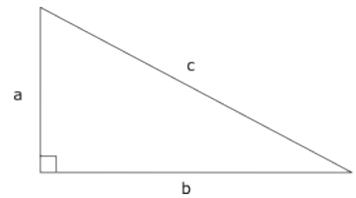
MATHEMATICS ENRICHMENT CLUB.
Solution Sheet 10, July 25, 2016

1. Since each 3×4 and 4×3 rectangle needs to have at least one black square, the minimum possible is 12. This can be achieved with the following configuration.

		X				X			X		
					X						
		X							X		
					X			X			
			X								
		X				X			X		

2. By substituting the two points $A(-2; 1)$ and $B(2; 9)$ into the equation of the parabola,



4. Let a and b be the shorter two sides of the triangle and c be the hypotenuse. Then we have

$$\frac{1}{2}ab = 3(a + b + c).$$

Dividing both sides by 3, using $c = \sqrt{a^2 + b^2}$ and rearranging

$$\frac{ab}{6} - (a + b) = \sqrt{a^2 + b^2}.$$

Squaring both sides,

$$\frac{a^2b^2}{36} - \frac{ab}{3}(a + b) + (a + b)^2 = a^2 + b^2.$$

Simplifying,

$$a^2$$

(d) $x - y = 7$ and $x^2 + xy + y^2 = 7$

The first and third cases have no solutions, the second case has solutions $\{x = 5; y = 6\}$; $\{x = -6; y = -5\}$ and fourth case has solutions $\{x = 3; y = -4\}$; $\{x = -4; y = 3\}$.

6.

Senior Questions

1. For $p = 2$ we have $2^2 + 2^2 = 8$ which is not prime. For $p = 3$, we have $2^3 + 3^2 = 17$ which is prime. For $p > 3$ (odd), we claim that $2^p + p$