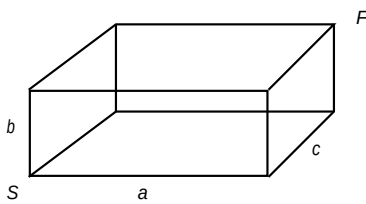


MATHEMATICS ENRICHMENT CLUB.
Problem Sheet 6, June 5, 2016

1. A spider, S is in one corner of a cuboid room, with dimensions $a \times b \times c$, and a fly, F is in the opposite corner; see figure below. Find the shortest distance from S to F (Note that spiders can't fly).



2. Working from left to right in a number, if the next digit is greater in value than the preceding digit, we say that the digits are strictly increasing; For example, 123, 247 and 367 are all 3-digit numbers with this property.
 Given a number has strictly increasing digits, what is the probability that it contains 5-digits?
3. How many ways are there to place one white king and one black king on an empty chessboard, such that they cannot attack each other?
4. Prove that $6^n + 8^n$ is divisible by 7 if and only if n is odd.
5. $ABCD$ is a parallelogram; X is the point on the diagonal BD . A line through X parallel to AB intersects AD at the point P ; a line through X parallel to BC intersects AB at Q . Show that the area of the quadrilateral $APCQ$ is half the area of $ABCD$.
6. Let x be a real number. Denote by $[x]$ the largest integer less than or equal to x ; For example, $[3] = 3$. Find all positive real numbers x, y satisfying the equation

$$[x][y] = x + y:$$

Senior Questions

1. In a chess tournament, every participant played with each other exactly once, receiving 1 point for a win, 1=2 for a draw and 0 for a loss. Is it possible that for every player P , the sum of points of the players who were beaten by P is greater than the sum of the points of the players who beat P ?
2. At the end of the school year it became clear that for any arbitrarily chosen group of no less than 5 students, 80% of the marks "A" received by this group were given to no more than 20% of the students in the group. Prove that at least 3=4 of all "A" marks were given to the same student.
3. Given the Fibonacci sequence 1; 1; 2; 3; 5; 8; 13; :: defined by the second order recurrence relation, $F_{n+2} = F_n + F_{n+1}$