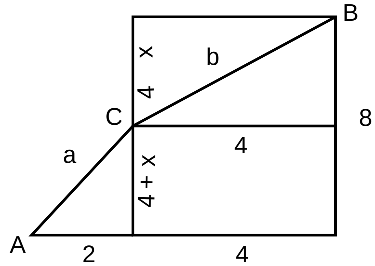
3. Let a be the greatest number written on a blackboard, pick another integer b on the board, then a b. Furthermore, there is an integer n 0 such that 2^n $a < 2^{n+1}$, so that $2^n < a + b$ $2a < 2^{n+2}$. Since $a + b$ must be a power of two, we must have $a + b$;m[5t hav0w85431f 84 -4.339 Td n(b)]TJ/F17 7.9701 Tf 5.138 0 Td [1+2

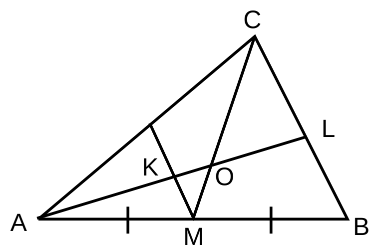
2 =16 + 4=32 + n-digit
rs contributes to the sum by an amount of $(n$ $1)!$ $(1 + 2 + 3 + ::: + n)$.

ke a similar argument by xing the second last digit, so that the second all combinations of n-digits long numbers contributes to the sum by an
n $1)!$ $(1 + 2 + 3 + ::: + n)$ 10.

his for all digits, then the total sum is $(n$ $1)!$ $(1 + 2 + 3 + ::: + n)$
$^2 + ::: + 10^n)$.



4. Consider the above diagram. By Pythagoras, $a = \sqrt{(4+x)^2 + 4}$ and $b = \sqrt{(x-4)^2 + 16}$; the two roads that connects the point A to B has length a and b. The total length of the roads; that is $a + b$ is minimised when the two lines AC and CB in the diagram are co-linear. Therefore, the combine length of the two roads is $\sqrt{(2+4)^2 + 8^2} = 10$.



5. (a) Since the lines KM and CB are parallel, the triangles $\triangle KMO$ and $\triangle OLC$ are similar. In particular, by angles and ratios we have the formula

$$\frac{|KO|}{|KL| - |KO|} = \frac{|OM|}{|MC| - |OM|}.$$

   (b) Since M is the midpoint of AB and the line KM, CB are parallel, by the midpoint theorem K is the midpoint of AL. Additionally, by using the fact that $2|MC| = |AL|$, we have $|KL| = |MC|$. Now substituting $|KL| = |MC|$ into the formula from part (a), we obtain $|KO| = |OM$. Therefore the triangles $\triangle KMO$ and $\triangle OLC$ are isosceles. Finally, using the condition $\angle OLC = 45^\circ$ we have $\angle COL = 90^\circ$.

6. Since the constant coefficient of $p(x)$ is 3, $abcd = 3$. Therefore,

$$\frac{abc}{d} = \frac{3}{d^2}, \qquad \frac{acd}{b} = \frac{3}{b^2}, \qquad \frac{abd}{c} = \frac{3}{c^2}, \qquad \frac{bcd}{a} = \frac{3}{a^2}.$$

Let $y = 3/x^2$, then $p(\sqrt{3/y}) = 0$ when $p(x) = 0$. Therefore rearranging $p(\sqrt{3/y}) = 0$ gives a polynomial of y with the required roots.