MATHEMATICS ENRICHMENT CLUB.

Hint Sheet 12, June 24, 2014 [1]

1. (a) Write 30! in its prime factorisation and divide by $10^7 = 2^7 \quad 5^7$.

   (b) Think about 1000!'s prime factorisation - how many pairs of 2s and 5s are there?

2. Each step removes 1 =4 of the current amount of red. Extra: What does this mean in the limit as n ! 1 ? Does this imply there's no red left?

3.

What if you get the guests to move to a room double their room number, so the guest in room 1 moves to room 2, then guest in room 2 moves to room 4 and so on. Now all the odd numbered rooms are free and we can accomodate an in nite number of guests.

(b) Think about this in two ways. If the probability is \the number of perfect squares divided by the number of numbers", then the probability is one. Every positive integer can be matched with one and only one perfect square by squaring it, so there are the same number of perfect squares as there are numbers (they are paired up exactly one-to-one).

But what if we look at the probability of choosing a perfect square less than n, well the number of perfect squares less than or equal to n is $b\sqrt{n}c$ and the number of numbers less than or equal to n is n. so $p < 1=\sqrt{n}$ which goes to 0 as n tends to in nity.