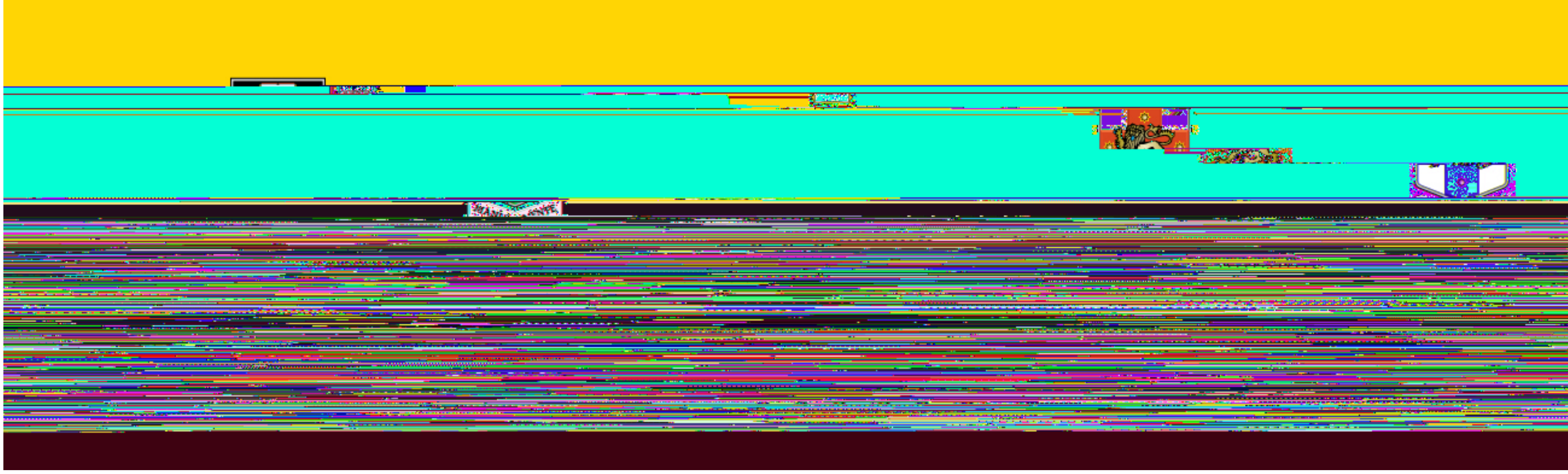

MATHEMATICS ENRICHMENT CLUB.[1]
Solution Sheet 8, June 25, 2013

1.

$$\begin{aligned}(2+4+6+\cdots+200)-(1+3+5+\cdots+199) &= (2-1)+(4-3)+(6-5)+\cdots+(200-199)\\ &= \underbrace{1+1+1+\cdots+1}_{100\ \text{terms}}\\ &= 100\end{aligned}$$

2. We must have a boy-girl-boy-girl configuration, of which there are two possibilities (starting with a boy or starting with a girl). Then within those configurations there are 6! possible ways to order the boys, and 6! possible ways to order the girls, making a total of $2 \times 6! \times 6!$ possible orderings.

3. There may have been some confusion with this question. A colouring here means painting each face with a colour from a selection of n colours (so we need not use all available colours) and two colourings are considered equivalent if you can rotate the tetrahedron of the first colouring and get the identical appearance of the second. So with one available colour, there is only one way to colour the tetrahedron - colour all the faces with the one colour.

   (a) Let's say the two colours are red and blue. There's the one colouring each for all red and all blue. Then we can have 3 faces red, 1 blue or 2 faces red, 2 blue or 1 face red, 3 blue. Of ach of the latter colourings there is only one. So finally there are 5 colourings.

   (b) Now we have red, blue and green. First, there's the 3 single colour colourings, all red, all blue, all green. Then for each pair of colours there was three colourings using exactly two colours, so thats 3 by red-blue, 3 by red-green and 3 by blue-green. Finally, fix the base as one colour, and the other 3 faces one colour each - there are 3 of these (one for each base colour). Try making more colourings by flipping an existing colouring and convince yourself you can always rotate back to where you started! So that's 15 in total.

[1]Some of the problems here come from T. Gagen, Uni. of Syd. and from E. Szekeres , Macquarie Uni. Question 5 comes from the Futurama episode "The Prisoner of Benda".

(c) Now we have red, blue, green and yellow. There's 4 single colour colourings. Then we have 6 pairs of colours, so that's 3 6 colourings with exactly 2 colours. Then there's 4 ways of choosing 3 colours, so that's 4 3 colourings with exactly 3 colours. Then colouring each face a di erent colour, there are only 2 of those (think about ipping the tetrahedron). That makes 36 all up.

4. (a) Triangles ABC and CDB are similar so $\frac{BC}{}$