

MATHEMATICS ENRICHMENT CLUB.  
Problem Sheet 9, July 23, 2013

1

1. The sequence  $a_1; a_2; a_3; \dots$  is arithmetic. If  $a_1 = 10$  and  $a_{a_2} = 100$  what is  $a_{a_{a_3}}$ ?
2. We play a game in which we try to get from one number to another. Each move we can replace the natural number  $n$  with  $ab$  if  $a + b = n$  and  $a$  and  $b$  are both natural numbers. Can we get to 2001 from 22 in any number of moves?
3. How many digits does the number  $125^{60}$  have?
4. Commander Keen is standing at the top left corner of an  $n \times n$  grid, but wants to get to the bottom right corner. He's only allowed to move to the right, or downwards.
  - (a) Draw all the possible paths from the top left to the bottom right if the grid is  $2 \times 2$ .
  - (b) How many possible paths are there if the grid is  $20 \times 20$ ?
  - (c) What about  $n \times n$ ?
5. Each of the six vertices of a regular hexagon are connected to every other vertex using either a red or a blue line. Show that, however this is done, the resulting diagram will always contain either a red or a blue triangle. Show that this is not always the case if we use the vertices of a pentagon.
6. Consider the two sequences  $x_0 = 1; x_1 = 1; x_{n+1} = x_n + x_{n-1}$