

MATHEMATICS ENRICHMENT CLUB.<sup>1</sup>  
Problem Sheet 6, June 11, 2013

1. The product of the ages in years of two adults is 770. What is the sum of their ages?
2. An automatic card shuffler always re-arranges the cards in the same way. The cards begin in the order A,2,3,4,5,6,7,8,9,10,J,Q,K and after 2 shuffles the order is 6,5,K,10,Q,8,2,3,7,J,9,A,4. What order do we get if we shuffle them three times?
3. (a) Show that the median to the hypotenuse of a right-angled triangle has length exactly half the length of the hypotenuse.  
(b) Let  $A; B; C$  be a triangle with  $A_1; B_1; C_1$  the midpoints of the sides  $BC; CA; AB$  respectively. Let  $D$  be the foot of the perpendicular from  $A$  to  $BC$ . Show that  $B_1C_1D$  is congruent to  $B_1C_1A_1$ .
4. Find all positive integers  $m$  and  $n$  such that  $3m - 1$  is a multiple of
  - (a) Find  $(12); (30)$ ;
  - (b) Suppose  $p$  is prime, find  $(p); (p^2); (p^3)$ ;
  - (c) If  $p$  and  $q$  are two different primes, find  $(p)$ .
6. Suppose  $S$  is the intersection of the three medians drawn through  $S$  parallel to  $BC$  meeting  $AC$  at  $T$ . Find the ratio of  $AST$  to the area of  $ABC$ ?

Senior Questions

1. Suppose that  $n$  is an odd integer greater than 1. Find the number of negative (real) roots of  $2x^n - nx^2 + 1 = 0$ :

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<sup>1</sup>Some of the problems here come from T. Gagen, Uni. of Syd. and from E. Szekeres, Macquarie Uni.

