MATHEMATICS ENRICHMENT CLUB.[1]
Problem Sheet 1, May 7, 2013

1. Laurie sold two cars for $25 000 each. One he sold at a 20% pro t and the other at a 20% loss. How much did he gain or lose ?

2. A number n has exactly 12 divisors. Given that n is divisible by 1,2,3,4,5 and 6 nd a possible value of n. Is it the only one? Explain.

3. Without using a calculator, which is larger 31^{24} or 257^{15}:

4. Let $S_n = 2n(2n \quad 1)(2n \quad 2)\cdots(n+1)$. For example, $S_3 = 6 \quad 5 \quad 4 = 120$.

   (a) What is the power of 2 in the prime factorisatoin of S_n for $n = 2; 3; 4\cdots$?

   (b) Make a conjecture based on(i) and prove it.

5. Without using a calculator, show that

$$\sqrt[3]{5\sqrt{13}+18} \quad \sqrt[3]{5\sqrt{13} \quad 18} = 3:$$

   (Hint: Let $x = a \quad b$ and cube.)

6. Let ABC be a triangle and $D; E$ points on $AB; BC$ respectively, and S be the intersection of AE and CD. If $AD = DB$ and $BE : EC = 2 : 1$, nd the ratios $CS : SD$ and $AS : SE$:

7. (a) Let P be an interior point in an equilateral triangle ABC. Prove that we can always form a triangle with sides of length $AP; BP; CP$. (That is, we have to show that the sum of any two of these lengths is larger than the remaining one.)

   (b) Give an example of a triangle and point inside it for which the above result is not true.

[1] Some of the problems here come from T. Gagen, Uni. of Syd. and from E. Szekeres , Macquarie Uni.