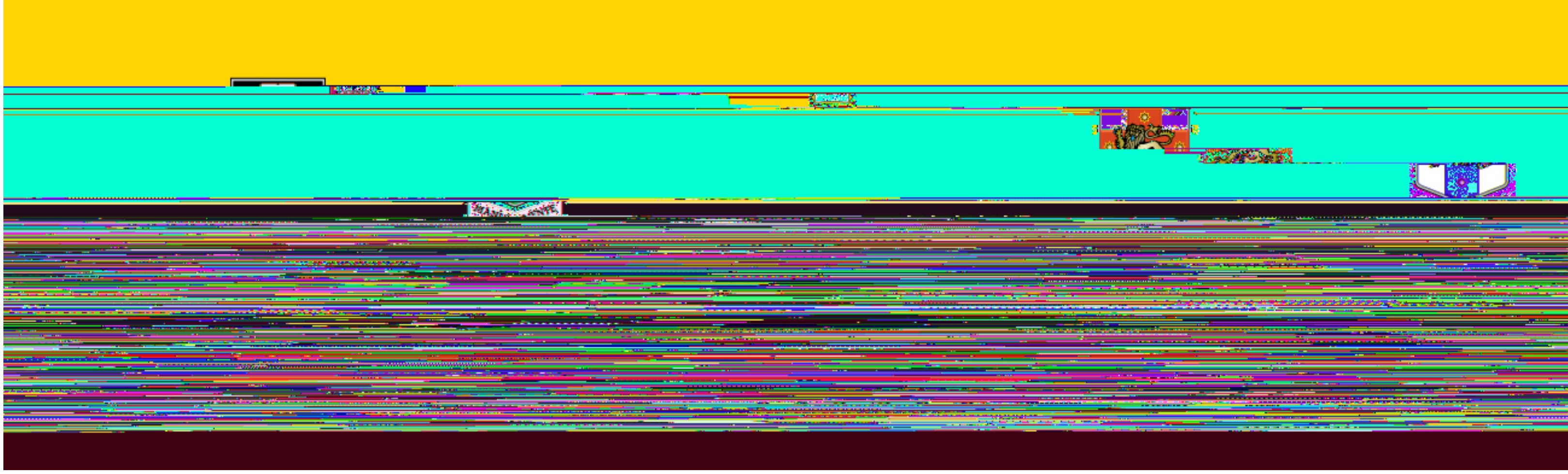


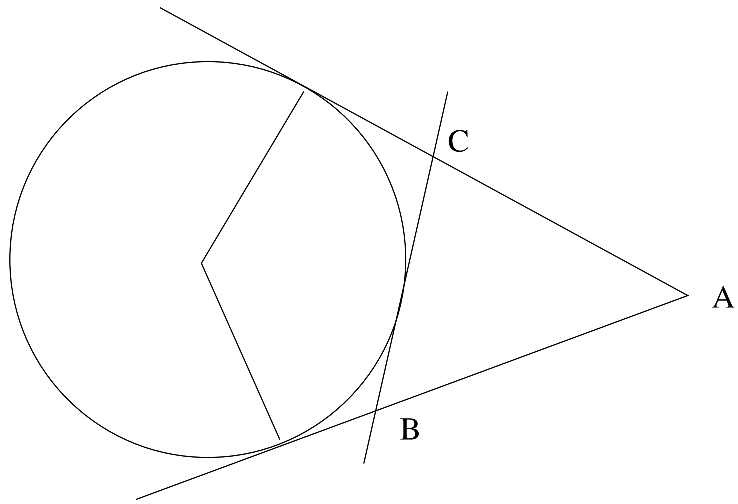
MATHEMATICS ENRICHMENT CLUB.[1]
Problem Sheet 16, September 10, 2012

1. A box of apples costs \$4, a box of oranges costs \$3 and a box of lemons costs \$2. A person buys 8 boxes of fruit at a cost of \$23. If at least one box of each kind of fruit is bought, find the largest possible number of boxes of apples.

2. Suppose a, b are positive real numbers. Use the diagram below to give a **geometric proof** that $a^2 = (a-b)^2 + 2ab - b^2$.

(b) Paul then measured the edges to be 2,3,4,5,6,8. If $AB = 2$ what is the length of CD?

7. A circle is drawn which touches BC in triangle ABC and also touches the two sides AB and AC produced at T and S respectively. Let O be the centre of this circle.



(a) Explain why OB bisects the angle TBC.

(b) Prove that the length of AT equals half the perimeter of the triangle ABC.

0.1 Senior Questions.

1. Prove that
$$\cos((n+2)\theta) = 2\cos((n+1)\theta)\cos\theta - \cos(n\theta),$$
for each integer $n \geq 0$.
Hence express $\cos 5\theta$ in terms of powers of $\cos\theta$.

2. For every positive real number $n > 1$, prove that
$$2\sqrt{n+1} - 2\sqrt{n} < \frac{1}{\sqrt{n}} < 2\sqrt{n} - 2\sqrt{n-1}.$$

3. Use the result in Q2 to prove that
$$2(\sqrt{N+1} - 1) < \sum_{n=1}^{N} \frac{1}{\sqrt{n}} < 2\sqrt{N}$$
and deduce that the sum of the first million terms of
$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \ldots.$$
is between 1998 and 2000.