

MERE9453

A UghYfg Dfc YWh7

HYfa "Hk c"#"&\$&%

7ci fgY'Cj Yfj]Yk

GHJZ:7cbHJWh:8YHJ]`g

7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`]lm	@cWJhjcb	D\cbY
7 Uf`]rc`HUVY`]b	WhUVY`]b4 i bgk "YXi "Ui		C`X'AU]b 6i]`X]b[<i>ž'@</i> Yj Y` %Fcca'%) - 7	Ž*%'f&Ł'-',) +-(*
M `>]b[mi"/]b[4 i bgk"YXi"Ui		F cca '&% ž HmfYY'9bYf[m HYVX.bc`c[]Yg 6i]`X]b['?!<*	Ž*%1&!-',) \$+,-

Hi hcfg

B Ua Y	9a U]`	5j U]`UV]`]m	@cWJhjcb	D\cbY
NU_\Uf [*] @UbYfW	n"`UbYHW4 i bgk "YXi "Ui	J]UʻHYUa gž'9a U]`cf D\cbY	HmfYY'9bYf[m HYW\bc`c[mi?! <*'&*%K G\$%	\$(*,(&,++,

GWXcc`7cbHJWhi±bZcfa Uh]cb

GW.cc`cZA]bYfU`g'UbX'9bYf[mFYgcifWygC`X'AU]b'6i]`X]b[ž@YjY`'%ž%)-'fP%)Ł IBGK 'GM8B9MBGK'&\$)&5IGHF5@=5

9b[]bYYf]b[]Gh XYbhGYfi]WYg

9. a YfY"hYUW]b[4 i bgk "YXi "Ui

K . 'k k k "Yb[]bYYf]b["i bgk "YXi "Ui #a]bYfU`g!YbYf[mfYgci fWyg

7ci fgY [.] 8YHJ]`g	
7fYX]h'Dc]bhg'('	
Gi a a UfmcZh\Y'7ci fgY	
H\Y`dfcYWndfcj]XYg`Ub`cddcfhib]hmZcf`mci`hc`Vf]b[`hc[Yh\Yf`Yb[]bYY	f]b['df]bW]d`Yg``YUfbYX'cj Yf

HYUWX]b['GhfUhY[]Yg'

 $H\Y'Wci fgY']g'HUi [\h'Ug'Ub']bX]j]Xi U''fYgYUfWX'dfc YWfz'hc'XYj Y`cd'U''Yj Y`'cZfYgYUfWX'g_]``g'UbXUi hcbca m'' \\$

Df]j UhY Ghi Xm

5ggYgga Ybh

5ggYgga YbhHUg_g

5ggYgga Ybh'tUg_	K Y][\h	8i Y'8UHY	Gh XYbh@YUfb]b[Ci hWca Yg 5ggYggYX
:]bU``H\Yg]g`	*\$1	\$*#\$, #&\$&%'%%) - 'DA	%ž'(ž')
DUffjWjdUfjcb'A Uf_') ।	Bch5dd`]WUV`Y	%ž'&ž'' ž'(ž')
F YgYUfWX DfYgYbHJHjcb	%) ၊	\$*#\$, #&\$&%`%%) - `DA	%ž'&ž'' ž'(ž')
H\Yg]g'5'A Uf_'	%\$I	Bch5dd`]WUV`Y	
H\Yg]g`6`A Uf_`	%\$I	Bch5dd`]WUV`Y	

5ggYgga Ybh8YHJ]`g

5ggYgga Ybh'% :]bU`H\Yg]g

8YHJ]`g.

 $\label{thm:lower} $$HY':]bU''HYg]g'WcbHJ]bg'U'Wca d'YhY''hYg]g'fZcfa UHjb['UbX'YI Ya d'Ufg'Zci bX'cb'A]WfcgcZh'HYUa gŁ'''HYY HYG]g'g\ci 'X']bWl XY.$

8cWa Ybh	• F Ydcfhcf'g`]XY ghfi WhifY UbX `Umci h	
dfYgYbhJhjcb	• 9b[`]g\'g_]``g'-'gdY``]b[ž'[fUa a Uf	
	8 UHU dfYgYbHUHcb f]ZUdd`]WUV`YŁ	
	• 7 `Uf]micZk f]n]b[
	• 7]huhcbg Webg]gh/bhUbX WeffYWhmZefa Uth/X	

5 https://bxubwww.Fyei]fYaYbhg

Ghi XYbhgʻUfYʻghfcb[`miYbWci fU[YX'hcʻUhhYbX'U```WUggYgʻUbX'fYj]Yk '`YWhi fY`fYWcfX]b[g"

7 ci fgY GWX YXi `Y

J]Yk 'WUgg'ha YHUV'Y

H]a YtUV`Y

8UhY	HmdY	7 cbhYbh
K YY_"%" %"A Um"!"(">i bY		GHJfhcZhYfa a YYh]b[k]h\gidYfj]gcfffgi[[YghYXŁ
K YY_').'&, '>i bY'!'&'>i `m		A]X!hYfa 'a YYh]b['k]h\'mci f'gi dYfj]gcf'fgi [[YghYXŁ
K YY_', .'%-'> `m!'&' '> `m		Gi Va]hXfUZnH\Yg]g'hc'gi dYfj]gcf'Zcf'ZYYXVUW_ fgi [[YghYXŁ
K YY_"%\$."&"5i [i gh!"*	5ggYgga Ybh	GiVa]hFYgYUfW.DfYgYbHJhjcbhcA]WfcgcZnHYUag
5i[igh	5ggYgga Ybh	Gi Va]h:]bU``H\Yg]g'j]U'Hi fb]h]b

Fygci fWyg

DfYgWf]VYX F Ygci fWYg

Ghi XYbhgʻUfYʻYl dYWhYXʻhc.

- FYUX hcd]Wa UhYf]U``dfcj]XYX Vmih\Y]f`gi dYfj]gcfj
- FYUX'a UlYf]U`g`dfcj]XYX`VmWti fgY`WtcfX]bÜlhcfz`]bWi X]b[`fi V]Wgz`Zcfa UlHjb[`[i]XYg`UbX YI Ya d`Ufg"

•

9i fcdYUb~5ggcWJUhjcb~cZ; YcgWJYbhjghg/ YU[Y"cf[_

9b[]bYYfg

DYlfc`Yi a '9I d`cfUl]cb'GcW]YlmicZ5i glfU`]U <u>dYgU"Wta "Ui</u>

GcW/YmcZ9IdcfUh/cb; Ycdmg/W/ghg gY["cf[

GcWymcZDYhfc`Yi a '9b[]bYYfg`fGD9Ł gdY"cf[_

GcWJYhmicZDYhfcd\mg]WJghg:/ "K Y```@c['5bU`mghg' gdk `U"cf[_

; YcgWJYbWY '5i glfU`]U [<u>U"[cj "Ui</u>

7ci fgY 9j U`i Uh]cb UbX 8Yj Y`cda Ybh

: YYXVUW_`Zfca 'ghi XYbhg']b'k Y`Wca YXž'UbX']g'i gYX'hc'Wcbh]bi ci g`mi]a dfcj Y'h\Y'Wci fgY ci hWca Yg'UbX'Yl dYf]YbW'g'Zcf'ghi XYbhg"

@UVcfUhcfmK cf_g\cd \dag{b}Zcfa Uhjcb

 $\label{localized-final} D`YUgY`WcbHJWimci f`gi dYfj]gcf`Zcf``UVcfUhcfm`Yei]da Ybhž`UVWWgg`cf`ch\Yf`gdYVJU``fYgci fW'g`mci `a][\hbYYX`hc`Wca d`YhY`mci f`h\Yg]g"$

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

H\Y'GW\cc``\Ug'XYj Y`cdYX'U'[i]XY`]bY'hc'\Y`d'mci'k\Yb'gi Va]hh]b['U'Wci fgY'Ugg][ba Ybh'

K Y YbWci fU[Y'mci'hc'fYhJ]b'U'WcdmcZYj YfmiUgg][ba Ybhgi Va]hYX'Zcf'UggYgga YbhZcf'mci f'ck b'fYWcfX Y]hYf']b'YUfXWcdmcf'Y'YWfcb]WZcfa"

5```UggYgga Ybhg'a i gh\Uj Y'Ub'UggYgga YbhWcj Yf'g\YYhUHUW\YX"

7ci fgY Wca d`Yhjcb

7 ci fgY 'Wca d`Yh]cb 'fYei]fYgʻgi Va]gg]cb 'cZU```UggYgga Ybh]hYa g": U]`i fY 'hc 'gi Va]h'U```UggYgga Ybh]hYa g a UmfYgi 'h']b 'h\Y 'Uk UfX 'cZUb'I bgUh]gZUWrcfm: U]`i fY 'fI : Ł'[fUXY 'Zcf'h\Y '7 ci fgY 'i b`YggʻgdYW]U` Wcbg]XYfUh]cb '\UgʻVYYb 'gi Va]h\YX 'UbX 'Uddfcj YX"'D`YUgY 'bchYž'U 'Wca dYhYbWm\i fX`Y 'cZ) \$i i]gʻUdd`]YX 'hch\Y`Z]bU``UggYgga Ybh'

@UhY Gi Va]gg]cb cZUb 5gg][ba Ybh

• I BGK "@YUfb]b["7 YblfY"!"

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

Mci f`YWi fYf'UbX'h\Y'I b]j Yfg]lmk]``YI dYWimci f`gi Va]lhYX'Ugg][ba Ybhg'UfY'lfi `mmci f`ck b'k cf_"'I BGK \Ug'j YfmWYUf'[i]XY`]bYg'cb'k \Uhd`U[]Uf]ga ']g'UbX'\ck 'hc'Uj c]X']h'D`U[]Uf]ga ']g'i g]b['h\Y'k cfXg'cf]XYUg'cZch\Yfg'UbX'dfYgYbh]b['h\Ya 'Ug'mci f`ck b"'D`U[]Uf]ga ']g'U'mdY'cZ]bhY``YWi U`'h\YZh''±i\Wb'hU_Y a UbmZcfa gžZfca 'XY`]VYfUhY'W\YUh]b['hc'UW]XYbhU``mWcdm]b['Zfca 'U'gci fW'k]h\ci hUW_bck `YX[Ya Ybh' H\Y'I b]j Yfg]lm\Ug'UXcdhYX'Ub'YXi WUh]j Y'UddfcUW\ 'hc'd`U[]Uf]ga 'UbX'\Ug'XYj Y`cdYX'U'fUb[Y'cZ fYgci fW'g'hc'gi ddcfhghi XYbhg"'5``h\Y'XYhU]`g'cb'd`U[]Uf]ga ž]bWi X]b['gca Y'i gYZ ``fYgci fW'gžWUb'VY Zci bX'Uhk k k "qhi XYbhi bak "YXi "Ui #d`U[]Uf]ga "

 $5 ```A]b]b["9b[]bYYf]b["ghi XYbhg"UfY"fYei]fYX"hc "Wca d`YhY"U"ghi XYbhXYWUfUh]cb" Zcf"UWUXYa]W]bhY[f]hm k \]W.]g"ci h]bYX']b"h\Y"Ugg][ba YbhWcj Yf"g\YYhg""6mg][b]b["h\]g"XYWUfUh]cbž"mci "U[fYY"h\Uh'mci f"k cf_"]g mci f"ck b"cf][]bU"k cf_"$

⊋mci 'bYYX'gca Y'UXX]h]cbU``gi ddcfhk]h\ 'mci f`k f]h]b['g_]``gž'd`YUgY'WcbhUWh\Y'@YUfb]b['7 YbhfY'cf'j]Yk gca Y'cZh\Y'fYgci fWg'cb'h\Y]f`k YVg]hY. 'k k k "`Wi bgk "YXi "Ui "'H\Y'@YUfb]b['7 YbhfY']g'XYg][bYX'hc'\Y`d mci ']a dfcj Y'mci f'UWUXYa]Wk f]h]b['UbX'Wca a i b]WUh]cb'g_]``g"'Gca Y'ghi XYbhg'i gY'h\Y'7 YbhfY'gYfj]W/g VYWUi gY'h\YmUfY'Z]bX]b['h\Y]f'Ugg][ba Ybhg'U'W\U``Yb[Yžch\Yfg'VYWUi gY'h\Ymk Ubhhc']a dfcj Y'Ub'U`fYUXm gi WW/ggZ ``UWUXYa]WdYfZcfa UbW/"

5 WUXYa]W±bZcfa Uhjcb

VfcUXVUbX`WcbbYWfjcb`f&)*`_V]h#gYWcf`ZJghYfŁ UV]`]hmhc'j]Yk 'ghfYUa]b['j]XYc'fN][\'cf``ck 'XYZ]b]h]cb'l BGK 'HJ 'cdh]cbgŁ

A cfY bzcfa Uhcb Uvci higmghYa 'fYei]fYa Ybhg]g Uj U] UV Y Wh<u>k k k "ghi XYbh"i bgk "YXI "Ul #a ccX Y!gmghYa!fYei]fYa Ybhg</u>

\$WWgg]b[7cifgYAUHYf]U`gH\fci[\AccX`Y

7 ci/fgY ci h]bYgžgi ddcfha UhYf]U`g`UfY i d`cUXYX`hc`A ccX`Yž'h\Y`i b]j Yfg]lmighUbXUfX`@YUfb]b[
A UbU Ya Ybh'GmghYa 'f@A GŁ" +b`UXX]r]cbžcb!`]bY`Ugg][ba Ybhigi Va]gg]cbg`UfY`a UXY`i g]b['h\Y
Ugg][ba Ybh'XfcdVcl 'ZJW]`]lmidfcj]XYX`]b`A ccX`Y"'5```Ybfc``YX`ghi XYbhg`UfY`Ui hca Uh]WU``m`]bWi XYX`]b
A dcXY`Zcf`YUW\`Wci fgY"'Hc`UWWgg`h\YgY`XcW`a Ybhg`UbX`ch\Yf`Wci fgY`fYgci fWrgž'd`YUgY`j]g]h`
k k "a ccX`Y"hY`h'i bgk "YXi "Ui

√ck KY7cbhUWhMci

\$\hina \gžh\Y`GW\cc``cf`mcif`WcifgY`WcbjYbcfg`a UmbYYX`hc`WcbhUWhmci`UVcihmcif`WcifgY`cf`mcif\pfc`a Ybh'\Ncif`WcifgY`WcbjYbcfg`k]```igY`h\Y`Ya U]``Z bWhcb`k]h\]b`A ccX`Y`cf`kY`k]```WcbhUWhmci`cbmcif'4 ghiXYbh'ibgk "YXi"Ui`Ya U]``UXXfYgg"

KY i \$XYfghUbXh\Uhmci a Um\UjY'Ub'Yl]gh]b['Ya U]``UVWci bhUbXkci X'dfYZYf'Zcf'mci f'l BGK 'Ya U]`g'hc

5W_bck `YX[Ya YbhcZ7ci bhfm

 $K_{Y'UW\underline{b}ck} YX[Y'hY'6YXY[U''dYcd'Y'k \c'UfY'hY'ffUX]h]cbU''W ghcX]Ubg'cZhY''UbXg'cb'k \]WX'I_BGK_?Ybg]b[hcb'WJa_di_g']g''cWJhYX'''$

5ddYbX]I . '9b[]bYYfg'5i ghfU`]U'f95Ł'DfcZYgg]cbU` '9b[]bYYf'7ca dYhYbWm GHJbXUfX

Dfc[fUa =bhybXYX=@YUfb]b['Ci hWca Yg	
?bck `YX[Y`UbX'g_]```VUgY	
D9%"%7ca dfY\Ybg]j Yž'h\YcfmVUgYX'i bXYfghUbX]b['cZh\Y'i bXYfd]bb]b['bUhi fU`'UbX d\mg]\WJ`'g\WJYb\Wg'UbX'h\Y'Yb[]bYYf]b['Z bXUa YbhJ`g'Udd`]\WJV`Y'hc'h\Y'Yb[]bYYf]b['X]g\WJd`]bY	
D9%"&"7cbWYdh U``i bXYfgHJbX]b['cZh\Y'a Uh\Ya Uh]Wgžbi a Yf]WJ``UbU`mg]gžgHJh]gh]Wgž'UbX Wca di hYf`UbX`]bZcfa Uh]cb'gW]YbWYg'k\]W\'i bXYfd]b'h\Y'Yb[]bYYf]b['X]gW]d`]bY	
D9%" '=b!XYdh\'i bXYfghUbX]b['cZgdYW]U`]ghVcX]Yg'cZ_bck `YX[Y'k]h\]b'h\Y'Yb[]bYYf]b[X]gW]d`]bY	
D9%'('8]gWfba YbhcZ_bck `YX[Y 'XYj Y`cda YbhUbX'fYgYUfWX 'X]fYWf]cbg'k]h\]b'h\Y Yb[]bYYf]b['X]gW]d`]bY	
D9%) '? bck 'YX[Y'cZYb[]bYYf]b['XYg][b'dfUWjWY'UbX'WcbhYl hi U`'ZJWcfg']a dUWjb['h\Y Yb[]bYYf]b['X]gWjd`]bY	
D9%"* I bXYfgHJbX]b['cZh\Y'gWcdYž'df]bW]d`Ygž'bcfa gž'UWVci bHJV]`]h]Yg'UbX'Vci bXg'cZ gi gHJ]bUV`Y'Yb[]bYYf]b['dfUWfJWY']b'h\Y'gdYW]ZJWX]gW]d`]bY	
9b[]bYYf]b['Udd`]WUh]cb'UV]`]hm	
D9&"%5dd`]WUh]cb`cZYgHUV`]g\YX`Yb[]bYYf]b[`a Yh\cXg`hc`Wca d`Yl `Yb[]bYYf]b[`dfcV`Ya gc`j]b[
D9&"&": `i YbhiUdd`]WUh]cb'cZYb[]bYYf]b['hYVXb]ei Ygžhcc`g'UbX'fYgci fWYg	
D9&" '5dd`]WUh]cb`cZgmghYa Uh]WYb[]bYYf]b[`gmbh\Yg]g`UbX`XYg][b`dfcWYggYg	
D9&"('5dd`]WUh]cb'cZgmghYa Uh]WUddfcUWXYg'hc'h\Y'WcbXi WhUbX'a UbU[Ya YbhcZYb[]bYYf]b[dfcYWng	
DfcZYgg]cbU`'UbX'dYfgcbU`'Ultf]Vi hYg	
D9' "%'9h\]WU`'WcbXi Wh'UbX'dfcZYgg]cbU`'UWWci bHJV]`]hm	
D9' "&'9ZYWAj Y'cfU`'UbX'k f]MYb'Wcaaib]WUAjcb']b'dfcZYgg]cbU`'UbX'`Um'XcaU]bg	
D9' "' '7 fYUhjj Yž']bbcj Uhjj Y'UbX'dfc!UWhj Y'XYa YUbci f	
D9' "('DfcZ/gg]cbU`i gY'UbX'a UbU[Ya YbhcZ]bZcfa Uh]cb	
D9' ") 'C fXYf'mia UbU[Ya YbhcZgY`Zi'UbX`dfcZygg]cbU``WcbXi Wh	
D9' "* '9ZZYWNjj Y'NYUa 'a Ya VYfg\]d'UbX'NYUa 'YUXYfg\]d	