

# PTRL3050

K Y```DfYggi fY'HYgh]b[

HYfa "H\fYY""##"&\$&\$

GW.cc``cZA ]bYfU`g`' 79bYf[ miF Ygci fWg'gb[ ]bYYf]b[ '#'' BGK ''#'' BGK ''9b[ ]bYYf]b

# 7 ci fgY<sup>-</sup>Cj Yfj ]Yk

### GHJZZ7cbHJWh78YHJ]`g

#### 7cbj Ybcfg

BUa Y	9a U]`	5j U]`UV]`]m	@cWUh]cb	D∖cbY
DYma Ub	dYmaUb4 i bgk "YXi"Ui			\$&-',))%&&
A cgHJ[ \]a ]				

#### @Y₩ni fYfg

BUa Y	9a U]`	5j U]`UV]`]m	@cWUh]cb	D∖cbY
5\aUX`8Y\[\Ub	U"XY\[\Ub_\U`]`]4 i bgk "YXi "Ui			
? \U`]`]				

#### 8Ya cbghfUhcfg

BUa Y	9a U]`	5j U]`UV]`]hm	@cWUhjcb	D∖cbY
HfU]k]h7\ib[	hfU]k]h'Wib[4ibgk"YXi"Ui			

GWXcc`7cbHJWhr=bZcfaUhjcb

# 7ci fgY<sup>·</sup>8YHJ]`g

#### 7 fYX]hDc]bhg\*\*\*

#### Gi a a UfmcZhXY7ci fgY

H\]gʻWcifgYʻk]```WcjYfʻh\YʻUbU`mf]WU``UbX`biaYf]WU``aYh\cXgʻhc`]bhYfdfYhkY```dfYggifY`hYghg"`H\YcfmcZ hfUbg]YbhkY```hYgh]b["`DfUWf]WU``UgdYWfgʻcZh\Y`XYg][b`UbX`dYfZcfaUbWY`cZZ]Y`X`hYgh]bghfiaYbhUh]cb" DfYggifY`XfUkXckb`hYghg"`:U``!cZZhYghg"`Ai`h]!fUhY`hYghg"`:UgʻkY```hYgh]b["`:`ck!UZhYf!Zck"`€gcW\fcbU``UbX ]gcW\fcbU``acX]ZYX"`=bhYfZYfYbWY`hYgh]b["`Di`gY`hYgh]b["`8f]``ghYa`hYghg":fUWfifYXfYgYfjc]f`hYghg" 5bU`mg]gʻcZai`h]d\UgY`Zck`hYghg"

#### 7 ci fgY 5]a g

H\Y`WcifgY`U]ag`hc`\Y`d`ghiXYbhg`]b`h\Y`XYg][b`cZhmd]WU``c]``UbX`[Ug`kY```Wcad`Yh]cb`UbX`]hg`]adUWficb fYgYfjc]f`dfcXiWhjj]hm'

H\Y`Wci fgY`]g`X]j ]XYX`]bhc`hk c`dUfhg"

DUfh' '% 5bU`mh]WU`aYh\cXg

z ``**7 \ UdhYf`%**Wcj Yfg`h\Y`[YbYfU``]bffcXi Wi]cb`hc`k Y```hYgh]b[žhmdYg`cZk Y```hYgh]b[žiZ bXUa YbHU``Zck Yei Uh]cbg`UbX`X]a Ybg]cb`Ygg`j Uf]UV`Yg`]b`XYgWf]V]b[`Zck `dfcdYfh]Yg''

z``**7\UdhYf`&**`Wcj`Yfg`Zck`]b`]XYU`]nYX`fYgYfjc]fgžX]ZYfYbhihmdYg`cZVci bXUfmWcbX]h]cbg`UbX WcffYgdcbX]b[`]XYU`]nYX`Zck`Yei Uh]cbg`Ug`kY``Zck`Yei Uh]cbg"

z <sup>··</sup>**7 \ Ud hY f**'' <sup>·</sup>Wcj Yfg <sup>·</sup>Ub <sup>·</sup>]bhfcXi Wijcb hc k Y<sup>···</sup>hYgh]bhYfdfYhUh]cb hYWkb]ei Ygži g]b[ <sup>·</sup>gYa ]!@c[ Uf]h\a ]W UbU`mg]gžhmdY!W fj Y a UhWk]b[ ždfYggi fY XYf]j Uhjj Y UbX dfYggi fY hYgh]b[ <sup>··</sup>]b fYU`]gh]WfYgYfj c]fg"

z <sup>...</sup>**7 \ UdHYf`(** <sup>.</sup>Wcj Yfg`H\Y`YZYWicZg\_]b`ZUWicf`UbX`k Y``VcfY`ghcfU[Y`cb`H\Y`dfYggi fY`HfUbg]YbHXUHU`UbX WcffYgdcbX]b[ ``]bY`gci fWY`gc`i Hcbgž`fYgYfj c]f``]a ]hHYgHjb[ 'UbX`]bHYfdfYHUHjcb`HYW\b]ei Yg`i g]b[ `gYa ]!@c[ d`chUbX`XYf]j UHjj Y`HmdY!`W fj Y`a UHW\]b[ 'HYW\b]ei Y"

z ``**7 \ UdhYf`)** 'Wcj Yfg'h\Y`XfUk Xck b`k Y```dfYggi fY`hYgh]b[ 'Zcf`j Uf]UV`Y`Zck `fUhYgž'WcffYgdcbX]b[ ]bhYfdfYhUh]cb`hYW\b]ei Yg'UbX`di `gY`hYgh]b[ 'XYg][ b`UbX`]bhYfdfYhUh]cb`a Yh\cXc`c[ ]Yg'' z ``**7 \ UdhYf**`\* `Wcj Yfg`]bhYfdfYhUh]cb`hYW\b]ei Yg`Zcf`dfYggi fY`Vi ]`X!i d`hYghžUdd`]WUh]cb`cZdfYggi fY XYf]j Uh]j Y`]b`Vi ]`X!i d`hYghUbU`mg]g`UbX`]bhYfdfYhUh]cb`cZVi ]`X!i d`hYghZc``ck ]b[`U`j Uf]UV`Y`Zck`fUhY"

 $z \stackrel{\cdot\cdot}{} 7 \underbrace{UdhYf'}{} + \underbrace{Wcj} Yfg'k Y \stackrel{\cdot\cdot}{} hYgh]bhYfdfYhUh]cb'hYWkb]ei Yg']b \underbrace{YhYfc[YbYci g'fYgYfj c]fg']bWi X]b['ZUWi fYX fYgYfj c]fg'k ]h gYU`]b['ZUWi fYX fYgYfj c]fg'k ]h gYU`]b['ZUWi fYX fYgYfj c]fg'k ]h gYU`]b['ZUWi hYK fYgYfj c]fg'k ]h gY$ 

DUfh & BiaYf]WU K Y DfYggi fY HYgh]b[

H\]gʻW\UdhYf`WcjYfgʻ%LibiaYf]WU`kY``hYgh]b[igb['ghUhY!cZh\Y!UfhWcaaYfW]U`g]ai`Uh]cb`gcZkUfYžig]b[ XUhU`Wcaacb`mUjU]`UV`Y`]b`]bXighfmUbX`&L`=bhYfdfYhUh]cb`cZibWcbjYbh]cbU``fYgYfjc]fg`kY```hYghg"

#### 7cifgY@YUfb]b[CihWcaYg

- % 5V]`]hmihc`[U]b'h\cfci[\`]bhYfdfYHUh]cb`cZVi]`X!id`UbX`XfUk!Xckb`kY```hYghg`Zcf`]bZ]b]hY`UWA]b[ fYgYfjc]fg`ig]b[`UbU`mh]WU``UddfcUVX/
- &" →XYbhJZmYUf`mUbX``UhY`hJa Y`YZYWbg`UbX`]bhYfdfYhUhJcb`cZfYU``k Y```hYghg
- '"=bhYfdfYhUh]cb`cZkY```hYghg`Zcf`\YhYfc[YbYcig`fYgYfjc]fg
- (" 5V]`]mihc`i gY`bi a Yf]WU``k Y```hYghiUbU`mg]g

#### HYUW ]b[ GhfUhy[ ]Yg

D`YUgY'fYZYf'hc'h\Y']bZcfaUh]cb']b'AccX`Y

# 5ggYgga Ybh

### 5ggYgga YbhHUg\_g

5ggYgga Ybh'HJg_	K Y][ \h	8iY'8UHY	GhiXYbhi@YUfb]b[ CihMcaYgʻ5ggYggYX
5gg][ba Ybhg	&\$1	Bch5dd`]WUV`Y	%ž <sup>:</sup> &ž <sup>:</sup> 'Ž (
: ]bU``9I Ua '	) \$I	8if]b[`h\Y`YIUa`dYf]cX	%ž 8,ž 'Ž (
A ]X HYfa '91 Ua '	' \$I	'\$#%\$#&\$&\$`\$(.\$\$`DA	%ž' &ž' † Ž' (

#### 5ggYgga Ybh8YHJ]`g

#### 5ggYgga Ybh'% 5gg][ ba Ybhg

GHJfhXUhY.Bch5dd`]WUV`Y

#### 8YHJ]`g.

GYjYfU``Ugg][baYbhg'Zcf'X]ZYfYbhgYWhjcbg'cZh\Y'WcifgY

#### 5ggYgga Ybh&:: ]bU``9l Ua

GhufhXUhY. Bch5dd`]WUV`Y

#### 8YHJ]`g.

: ]bU``YI Ua `fXi f]b[ `h\Y`YI Ua `dYf]cXŁ

5ggYgga Ybh' . A ]X HYfa '9I Ua

GhUfhXUhY.Bch5dd`]WUV`Y

#### 8 YHJ]`g.

A ]X hYfa 'Yl Ua

FYgcifWYg DfYgWf]VYX`FYgcifWYg`

# Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

H\Y'GW\cc``\Ug'XYjY`cdYX'U'[i]XY`]bY'hc'\Y`d'mci'k\Yb'giVa]hhjb['U'WcifgY'Ugg][baYbh'

K Y`YbWci fU[Y`mci `hc`fYHJ]b`U`WcdmcZYj YfmUgg][ba Ybhigi Va ]HYX`Zcf`UggYgga YbhZcf`mci f`ck b`fYWcfX Y]h\Yf`]b`\UfXWcdmicf`Y`YWfcb]WZcfa "

5```UggYgga Ybhg`a i gh\Uj Y`Ub`UggYgga YbhWcj Yf`g\YYhUhUVXYX"

### @UhY Gi Va ]gg]cb cZUb 5gg][ ba Ybh

: i ```a Uf\_g`Zcf`Ub`Ugg][ ba YbhUfY`cb`mdcgg]V`Y`k \Yb`Ub`Ugg][ ba Ybh`]g`fYWY]j YX`Vmh\Y`Xi Y`XUhY"`

K Y`i bXYfgHUbX`H\Uh'Uh'Ha Yg`mci `a Umbch'VY`UV`Y`hc`gi Va ]h'Ub`Ugg][ ba Ybh'cb`Ha Yž'UbX`H\Y`GV\\cc``k ]`` UVWca a cXUHY`UbmZU]f`'UbX`fYUgcbUV`Y`YI HYbg]cb"`K Y`k ci `X`fYVca a YbX`mci `fYj ]Yk `H\Y`I BGK `GdYV]U` 7 cbg]XYfUHjcb`[ i ]XY`]bYg`-gYY'gYWHjcb`VY`ck "

@UhY`gi Va ]gg]cb`k ]``bchVY`UVWYdhYX`UbX`k ]``VY`Wcbg]XYfYX`Ug`bc`gi Va ]gg]cb"

# GdYWJU``7cbg]XYfUh]cb

Mci 'WUb'Udd`mZcf`gdYWJU`'Wcbg]XYfUH]cb'H\fci [\`<u>H\Y'Bi WYi g'GH XYbh<iV</u>'k \Yb']``bYgg'cf'cH\Yf WJfW a gHUbWYg']bHYfZYfY'k ]H\`mci f'UggYgga YbHdYfZcfa UbWY'''G]W\_bYggž'a ]gUXj YbH fY'cf'cH\Yf WJfW a gHUbWYg'VYmcbX'mci f'WcbHfc`'a Um

- DfYj Ybhmci 'Zfca 'Wca d'Yhjb[ 'U'Wci fgY'fYei ]fYa Ybh
- ? YYd mci Zfca UthYbX]b[ UbUggYggUVY UVhjj ]hm
- GhcdimciigiVa]hh]b["UggYggUV"Yik cf\_"Zcf"U"WciifgY
- G][ b]]/WUbhmUZYWhinci f`dYfZcfa UbWF]b`UggYggUV`Y`k cf\_ž'VY`]hU`Zcfa U``YbX!cZgYa YghYf YI Ua ]bUh]cbz`U`WUgg`hYghz`U``UVcfUhcfmhYghz`U`gYa ]bUf`dfYgYbhUh]cb`cf`Ubmch\Yf`Zcfa `cZ UggYgga Ybh

K Y`Ug\_`h\Uhimci `d`YUgY`WebhUWih\Y`7 ci fgY`7 cbj Ybcf`]a a YX]UhY`mcbWY`mci `\Uj Y`Wea d`YhYX`h\Y`gdYW]U` Webg]XYfUh]cb`Udd`]WUh]cbžbc``UhYf`h\Ub`cbY`k YY\_`Zca `gi Va ]gg]cb"

A cfY`XYHJ]`g`cb`gdYWJU``Wcbg]XYfUH]cb`WUb`VY`Zci bX`Uh`<u>k k k "ghi XYbH'i bgk "YXi "Ui #gdYWJU`!Wcbg]XYfUH]cb</u>

# Gh XYbhGi ddcfh

H\YʻI b]j Yfg]miUbXʻh\Yʻ: UW `midfcj ]XYʻUʻk ]XYʻfUb[YʻcZgi ddcfhigYfj ]WgʻZcfʻgh XYbhgž']bWi X]b[.

- @]VfUfmilfU]b]b[ 'UbX'gi ddcfhigYfj ]Wfg'! '<u>k k k "`]VfUfm'i bgk "YXi "Ui</u>
- I BGK @YUfb]b[ '7 YbffY '! '<u>k k k "Wi bgk "YXi "Ui</u>
- 7 ci bgY``]b[ 'gi ddcfh! <u>k k k "Wci bgY``]b[ "i bgk "YXi "Ui</u>

9ei ]HUV`Y`@YUfb]b[`GYfj]WYg`U]a g`hc`dfcj]XY`U```ghi XYbhg`k]h\`U`ZYY`UbX`WcbZ]XYbh]U``gYfj]WY`h\Uh dfcj]XYg`dfUWf]WU``gi ddcfhhc`Ybgi fY`h\Uhmci f`\YU`h\`WcbX]h]cb`XcYgbfhiUXjYfgY`mUZYWhmci f`ghi X]Yg"` <u>\hhdg.#ghi XYbh'i bgk "YXi "Ui #Y`g</u>

# 5WUXYa ]W<cbYghmUbX`D`U[ ]Uf]ga

Mci f``YWi fYf`UbX`h\Y`I b]j Yfg]Imik ]``YI dYWinci f`gi Va ]IhYX`Ugg][ ba Yblg`UfY`Hi`minci f`ck b`k cf\_"`I BGK \Ug`j YfmWYUf`[ i ]XY`]bYg`cb`k \Uhd`U[ ]Uf]ga `]g`UbX`\ck hc`Uj c]X`]H`D`U[ ]Uf]ga `]g`i g]b[ `h\Y`k cfXg`cf ]XYUg`cZch\Yfg`UbX`dfYgYbh]b[ `h\Ya `Ug`nci f`ck b"`D`U[ ]Uf]ga `]g`U`mdY`cZ]bH``YWi U``h\YZh'' #i`WUb`HU\_Y a UbmZcfa gž`Zīca `XY`]VYfUHY`W\YUh]b[ `hc`UVW]XYbHU``mVcdm]b[ `Zīca `U`gci fWY`k ]h\ci hUW\_bck `YX[ Ya Ybh' H\Y`I b]j Yfg]Imi\Ug`UXcdhYX`Ub`YXi WUh]j Y`UddfcUV\`hc`d`U[ ]Uf]ga `UbX`\Ug`XYj Y`cdYX`U`fUb[ Y`cZ fYgci fWYg`hc`gi ddcfhigh XYbhg''`5```h\Y`XYHJ`g`cb`d`U[ ]Uf]ga ž`]bWi X]b[ `gca Y`i gYZ ``fYgci fWYgž`WUb`VY Zci bX`Uhi<u>k k k "gh XYbh'i bgk "YXi "Ui #d`U[ ]Uf]ga</u>

### 5WUXYa]W=bZcfaUh]cb

# 7 ci fgY FYgi `hg

: cf<sup>\*</sup>XYHJ]`gʻcbʻI BGK <sup>\*</sup>UggYgga Ybhdc`]Whž'd`YUgY`j ]g]h<u>'k k k "gh XYbH'i bgk "YXi "Ui #UggYgga Ybh</u>

=b`gcaY`]bgHJbWfg`mcif`Z]bU``WcifgY`fYgi`haUmVY`k]h\\Y`X`UbX`bchfY`YUgYX`cb`h\Y`IBGK`d`UbbYX`XUHY" H\]g`]g`]bX]WJHYX`VmU`WcifgY`[fUXY`fYgi`hcZY]h\Yf.

- @9'-']bX]WUhYg'mci '\Uj Y'bchWca d'YhYX'cbY'cf'a cfY']hYa g'cZUggYgga Ybh/cf
- K 8 -- ]bX]WUHYg'H\YfY']g'Ub']ggi Y'k ]H\ cbY'cf'a cfY'Ugg][ ba Ybh/cf
- K 7 '-- 'k \]WX ']bX]WUHYg'mci '\Uj Y'Udd`]YX'Zcf'GdYW]U`'7 cbg]XYfUH]cb'Xi Y'hc']``bYgg'cf'a ]gUXj Ybhi fY UbX'h\Y'Wci fgY'fYgi `hg'\Uj Y'bchVYYb'Z]bU`]gYX"

=b`Y]ħ\Yf`Yj Ybh`]ħk ci `X`VY`mci f`fYgdcbg]V]`]mħc WcbħUWħ\Y`7 ci fgY`7 cbj YbYf`Ug`gccb`Ug`dfUWfWUV`Y`Vi h bc``UħYf`ħ\Ub`Zj Y`f) Ł`XUmg`UZħYf`fY`YUgY`cZħ\Y`Wći fgY`fYgi `ħ`=Zmci `Xcb'ħWcbħUWħ\Y`Wcbj YbYf`cb`ħ]a Yž mci `a UmVY`fYei ]fYX`ħc`fY!gi Va ]ħUb`Ugg][ ba Ybħcf`fY!g]ħħ\Y`Z]bU``YI Ua `UbX`a UmfYgi `ħ`]b`mci `ZIJ`]b[ `ħ\Y

7F=7CG`Dfcj]XYf`7cXY.`\$\$\$-,;

### 5W\_bck `YX[ Ya YbhcZ7ci bhfm

 $\label{eq:constraint} \begin{array}{l} K = Y^{T}UW_{bck}^{T}X[Y^{T}KY^{T}GYXY[U^{T}dYcd^{Y}K \ c^{U}fY^{T}KY^{T}fUX]h]cbU^{T}W_{ghc}X]Ubg^{T}cZhY^{T}UbXg^{T}cbK \ NW^{T}=BGK \\ ? Ybg]b[hcb^{T}Wa = dig^{T}g^{T}cWUhYX'' \end{array}$