

ARTS1810

: ci bXUhcbg cZDc`]hWg UbX +bhYfbUhcbU`F Y`Uhcbg

HYfa 'C bY''# '&\$&\$

7ci fgY⁻Cj Yfj]Yk

'GhUZZ'7cbhUWh'8YhU]`g'

'7cbj Ybcfg'

B Ua Y	9a U]`	5j U]`UV]`]lm	@cWUh]cb	D∖cbY
B]Wkc`Ugʻ5dc]Z[g	b"Udc]Z]g4 i bgk "YXi "Ui	K YXbYgXUmgž'!(Fcca '%((-',)
			A cfj Yb 6fck b	

GWXcc``7cbHJWhʻ±bZcfa Uh]cb

GWXcc``cZGcWJU``GWJYbWYg

Fcca '%) -

A cfj Yb 6fck b 7 &\$

Ya U]`. 'gcgg4 i bgk "YXi "Ui

d\cbY. '\$&'-',) '% \$+

7ci fgY 8YHJ] g

7 fYX]hDc]bhg**

Gi a a UfmcZh\Y'7ci fgY

K \YfY XcYg'dck Yf']Y']b'dc`]f]Wg3'K \cgY'Yl d'UbUh]cbg'UVci h\ck 'h\Y'k cfX'k cf_g'UfY'j U'i YX'UbX'k \m8 H\]g'Wci fgY'k]``Yei]d'mci 'k]h\ gca Y'cZh\Y'g_]``g'bYW\ggUfmhc'VY[]b'Zcfa i `Uh]b['mci f'ck b'Ubgk Yfg'hc h\YgY'ei Ygh]cbg''Mci 'k]``'VY']bhfcXi WYX'hc'h\Y']XYU'cZgh Xm]b['dc`]h]Wgž'Vch\ 'Xca Ygh]WUbX']bhYfbUh]cbU'ž Ug'Ub''UWUXYa]WX\]gWJd`]bYž'UbX'hc'gca Y'cZh\Y'_YmUWUXYa]WWcbj Ybh]cbg'UbX'ghUbXUfXg'h\Uh'UfY]a dcfhUbhhc'h\Y'X]gWJd`]bYž'UbX'hc'gca Y'cZh\Y'_YmUWUXYa]Wwcbj Ybh]cbg'UbX'ghUbXUfXg'h\Uh'UfY]a dcfhUbhhc'h\Y'X]gWJd`]bY.'h\Y']XYU'cZYj]XYbW!VUgYX'Uf[i a Ybhž'ZcfYl Ua d'Yž'UbX'h\Y']XYU'cZYh\]WU' gW\c`Ufg\]d'''K Y'k]``'U`gc'X]gW gg'h\Y'Zcfa Uh]cb'cZh\Y'a cXYfb'ghUhY'gmghYa 'UbX']bhYffc[UhY'h\Y WcbWfdh U`]gUh]cb'cZh\Y'ghUhY'h\Uh]bZcfa g'a i W\'WcbhYa dcfUfmh\YcfmUbX'dfUWJ\Wf\]b'dc`]h]Wg'Uhh\Y bUh]cbU``Yj Y`''

5 gi VgrUbrjU``dUfficZh\]g Wci fgY`]g XYj chYX'hc'h\Y`]blfcXi Wjcb'cZh\Y'a U]b'h\YcfYhJW`'lfUX]hjcbg'h\Uh Ub]a UhY'h\Y'ghi XmcZdc`]hjWg'UbX`]bhYfbUhjcbU``fY`Uhjcbg'''C bY'cZh\Y'W'blfU``a YggU[Yg'cZh\Y'Wci fgY']g h\UhX]ZZYfYbhih\YcfYhJWD`'UddfcUW\Yg'[YbYfUhY'X]ZZYfYbhi]a U[Yg'cZh\Y'k cfX'h\UhiVi]`X'cb'dUffJW`'Uf Uggi a dhjcbg'''H\YfYZcfYžk \]'Y'dYcd'Y'a Umih\]b_'h\Ym_bck 'k\Uhih\Y'W ffYbhdfcV'Ya g'cZdc`]hjWg'UbX [`cVU`'dc`]hjWg'UbX'\ck 'hc'gc`j Y'h\Ya ž'cbY'cZh\Y'U]a g'cZh\]g'Wci fgY']g'hc'U`Yfhmci 'hc'ch\Yf'k Umg'cZ gYY]b['h\]b[g"'H\]g'g\ci 'X'U``ck 'mci 'hc'a U_Y'U'a cfY'WcbZJXYbhXYWJg]cb'UVci hmci f'ck b'glUbW'hck UfXg dUffJW`Uf']ggi Yg'UbX'hc'UbU`mgY'h\YgY']ggi Yg'a cfY'h\cfci [\`mž'Vi h]hig\ci `X'U``gc'a U_Y'mci 'ei Ygh]cb'Vch\ mci f'ck b'UbX'ch\Yfg''fYdfYgYblUhjcbg'cZh\Y'k cfX''

7ci fgY @YUfb]b[Ci hWca Yg

%"

dfYgYbhUh]cb`cZj]XYc`W]dgžga U``![fci d`UWhj]h]YgžE/5`gYgg]cbg`UbX`dU]f!kcf_"`H\Y``YWh fYg`UfYXYg][bYX`hc`gi dd`Ya Ybhi]bXYdYbXYbhgh Xmcb`h\Y`Wci fgY`h\UhVY[]bg`fVi hg\ci `X`bchYbXŁk]h\`WcgYYb[U[Ya Ybhik]h\`h\Y`kYY_`mfYei]fYX`fYUX]b[g"`@YWh fY`UhhYbXUbWY`]g`Wca di `gcfm!Vi h]hk]``VY`kY``kcfh\`]h

&""HI HCF =5@G

Hi hcf]U`g`U``ck imci ihc`X]gW gg`ei Ygh]cbgžh\Ya Yg`UbX`]ggi Yg`Uf]g]b[iZfca imci f`]bXYdYbXYbhighi Xmž`Ug`k Y`` Ugʻdfcj]X]b[ih\Y`cddcfhi b]hmhc`Yb[U[Y`]b`dfcXi Whj Y`X]gW gg]cb`k]h\ ih\Y]f`dYYfg`UbX`hc`dUfh]V]dUhY`]b`U fUb[Y`cZghfi Whi fYX``YUfb]b['UWhj]h]Ygž`]bWi X]b['[fci d#dU]f#bX]j]Xi U``UbU`mh]WU``k cf_žiZcfa U``UbX`]bZcfa U` XYVUhYgžei]nnYg`UbX`a]b]!dfcYWhg"`Hi hcf]U`g`UfY`U`gc`%\ci f`]b`Xi fUh]cb"

'"-₽89D9B89BH'@95FB₽; '57HJ +H-9G

±0XYdYbXYbh`YUfb]b['UWfj]hjYg'a i ghVY'Wca d`YhYX'YUW\k YY_'UbX'g\ci `X'hU_Y'Uddfcl]a UhY`m' \$!a]bg" H\YmUfY'XYg][bYX'hc'ZcghYf']bXYdYbXYbhighi XmUbX'hc'U``ck 'Zcf'Yb[U[Ya Ybhik]h\x]ZYfYbhHUg_g'h\Uh WUbbchVY'i bXYffU_Yb']b'U'hi hcf]U``Ybj]fcba Ybhžgi W\'Ug'WcbXi Wfjb['cb`]bY'fYgYUfW\'cf'k UhW\]b['U'Z]`a "H\YgY'UWfj]hjYg'UfY'WYUf'mYl d`U]bYX'cb'h\Y'l b]j Yfg]hm@A G'UbX'k]``a cghmVY'ZJWj]hUhYX'Vm]h'

H\]g`V`YbXYX`a cXY`cZXY`]j Yfm'Vch\`]bWfYUgYg`mci f`ZYI]V]`]lmž`Ug`mci `WJb`Wca d`YhY`dfYdUfUhcfm'UWfj]hjYg Uh'U`hja Y`h\Uhigi]hg`mci ž`UbX`XYYdYbg`Yb[U[Ya Ybhž`Ug`mci `k]```Uff]j Y`]b`h\Y`WUggfcca `k Y``!dfYdUfYX`UbX UV`Y`hc`i bXYfh'U_Y`Wca d`YI ž`dfcV`Ya !VUgYXž``YUfb]b[`UWfj]hjYg`]b`h\Y`ZUWY!hc!ZUWY`WUggYg"`

5ggYgga Ybh

: cf'U```]bZcfa Uh]cb'fY[UfX]b['I BGK 'GCGG'dc`]WmffYZYfYbW]b[ž'd`U[]Uf]ga ž'`UhYbYgg'YhW£'gYY.

 $\label{thm:condition} $$ \down{1.5} $$ \do$

DYbU`h]Yg`h\Uha UmVY`Udd`]YX`hc`mci f`k cf_.

• Mci "ghUbX"hc"

 $ZYYXVUW_g YYh\#i Vf]Wk] ``VY'Uj U] `UV'Y'hc'ghi XYbhg'Uhh\Y'ghUfhicZh\Y'Wci fgY'gc'h\Uhh\YmWUb'k cf_hck UfXg'gdYW]ZYX'ghUbXUfXg''$

 $\label{thm:continuous} \begin{tabular}{ll} Hi & fb & fb & gYM &$

5ggYgga Ybh' 9l Ua

GhUfh'XUhY. '\$\%\$) #&\$&\$ '\$-.\$\$ '5A

8 YHJ]`g. '9 | Ua `]b'th\Y'cZZWJU`'| BGK 'Y| Ua 'dYf]cX"'H\]g']g'th\Y'Z]bU``UggYgga Ybh]b'th\Y'Wti fgY"'Ghi XYbhg k]``fYWY]j Y'U'bi a Yf]WU``[fUXY'k]h\]b'th c'k YY_g'cZgi Va]gg]cb'th\fci [\'th\Y'| b]j Yfg]mfig'@YUfb]b[AUbU[Ya YbhGmghYa 'f@AGŁ"

5XX]h]cbU`XYhU]`g.

H\]g^]g^U^HJ_Y!\ca Y^Y! Ua ^UXa]b]ghYfYX^h\fci [\`mci f^A ccX`Y^dcfhJ`žXi f]b['h\Y^Zcfa U^I BGK ^Y! Ua dYf]cX"^H\Y^Y! Ua ^ei Ygh]cbg^k]^``Uj U]`UV`Y^Uh- Ua ^cb^: f]XUmh\Y^%ghcZA Umž&\$&\$"`H\Y^Y! Ua ^k]```VY gi Va]hhYX^h\fci [\`h fb]h]b#a ccX`Yž'UbX^]g^Xi Y^Vm- Ua ^A cbXUmh\Y^(h\`cZA Um&\$&\$"`A cfY`XYhJ]`g^UfY Uj U]`UV`Y^cb^A ccX`Y

 $\label{eq:higher} \begin{tabular}{ll} High \$

FYgci fWYg

DfYgWf]VYX F Ygci fWYg

gYY 'A ccX`Y

FYWca a YbXYX FYgci fWYg

GYY 'A ccX`Y

7ci fgY '9j U`i Uh]cb 'UbX '8Yj Y`cda Ybh

GYY 'A ccX`Y

5 WUXYa | W < cbYghmUbX D`U[| Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i±iWUbihU_Yia Ubm Zcfa qžZfca iXYi]VYfUhYiW\YUh|b[ihciUVW]XYbhU`mWcdm]b[iZfca iUiqci fWYik]h\ci hiUW_bck iYX[Ya Ybh'i

IBGK '[fci dg'd'U[]Uf]ga ']bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl h'cf']XYU'k]h\ci h'UW_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf_g"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWhgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k \YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'

⇒ Uddfcdf]UhY 'dUfUd\fUg]b[...7\Ub[]b['U'ZYk 'k cfXg'UbX'd\fUgYg'k \]'Y 'a cghmfYhJ]b]b['h\Y'cf][]bU` ghfi Wh fY 'UbX#cf'dfc[fYgg]cb'cZ]XYUg'cZh\Y'cf][]bU'žUbX']bZcfa Uh]cb'k]h\ci hUW_bck `YX[Ya Ybh'H\]g'U`gc Udd`]Yg']b'dfYgYbhUh]cbg'k \YfY'gca YcbY'dUfUd\fUgYg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci hWfYX]hUbX'hc d]YW]b['hc[Yh\Yf'ei chYg'UbX'dUfUd\fUgYg']bhc'U'bYk 'k \c`Yžk]h\ci hUddfcdf]UhY'fYZYfYbW]b["

7 c``i g]cb. 'k cf_]b['k]th 'cth Yfg'Vi hidUgg]b['cZZth Y 'k cf_'Ug'U'dYfgcb'g']bX]j]Xi U``k cf_"'7 c``i g]cb'U`gc]bWi XYg'dfcj]X]b['mci f'k cf_'thc'Ubcth Yf'ghi XYbh'VYZcfY'th Y'Xi Y'XUHYZcf'Zcf'th Y'di fdcgY'cZth Ya

- i bXYfgfUbX mci f'f][\hg'UbX'fYgdcbg]V]`]hJYg'Ug'U'gh XYbhUhl BGK
- VY`Uk UfY`cZd`U[]Uf]ga ž`Wcdmf][\hz'l BGK `Gh XYbh7cXY`cZ7cbXi WhUbX`5VWWdhUV`Y`l gY`cZ I BGK `=\forall H`F Ygci f\W\g`Dc`]\m\m
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY'qYfj |WYq'UbX']bZcfa UHcb'UVci hi BGK 'UbX'l BGK '@|VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci ž'ch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWng'cZ9@+69'k]``\Y`d'mci 'a U_Y'h\Y'a cghicZmci f'ghi X]Yg'UhI BGK "'\\hd.##qi VYWfi |XYg"`]VfUfm'i bqk "YXi "Ui #\`]qY#JVci hY`]qY

5 WUXYa]W = DZcfa Uhjcb

: cf`YggYbhJU``gh XYbh]bZcfa UhJcb`fY`UhJb['hc.

fYei YghgʻZcfʻYl hYbg]cb/ `UhYʻgi Va]gg]cbgʻ[i]XY`]bYg/ fYj]Yk ʻcZa Uf_g/ I BGK ʻ< YUʻh\ 'UbXʻGUZYhmdc`]WJYg/