

SRAP5103

F YgYUfWX F Ydcfh

HYfa "Hk c"#"&\$%

7ci fgY^{*}Cj Yfj]Yk

`GHJZZ:7cbHJWh:8YHJ]`g`

7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`] m	@cWUh]cb	D\cbY
8f'D\]``]dU'9j Ubg	d"Yj Ubg4 i bgk "YXi "Ui	6mUddc]bha Ybh VYZcfY cf'UZhYf gYa]bUfg'cf'Uhch\Yf bY[ch]UhYX'h]a Yg	F cca '(&'@Yj Y` ; ž'A cfj Yb 6fck b	-',);,% -

GW.cc`7cbHJWhi±bZcfaUhjcb

GWXcc``cZGcWJU``GWJYbWYg

Fcca '%) -

7ci fgY'8YW]`g

7 fYX]hDc]bhg *

Gi a a UfmcZh\Y 7ci fgY

±0 'h\]g'Wci fgYz'mci 'k]``\Uj Y'h\Y'cddcfhi b]hmhc'i bXYfhU_Y'U'd]YWY'cZX]fYWYX'cf][]bU`'fYgYUfW\'k]h\Y gi ddcfhicZUb'UWUXYa]WUXj]gcf"'Mci 'k]``Wca d`YhY'U'bi a VYf'cZZcfa Uhjj Y'hUg_gz'Vi]`X]b['i d'hc'h\Y gi Va]gg]cb'cZU'g]b[`Y'fYgYUfW\'fYdcfhiXYhU]`]b['h\Y'XYg][bz'WcbXi WhUbX'Z]bX]b[g'cZmci f'fYgYUfW\'"

, -' - 'A UghYf'cZ8Yj Y`cda Ybh'Gh X]Ygž', &&) 'A UghYf'cZ5fhg'f\bhYfbUh]cbU``F Y`Uh]cbgŁ'UbX', &(, 'A UghYf'cZ Dc`]Wn'Gh X]Yg'a i ghgYY_'Uddfcj U``Zfca 'h\Y'Dfc[fUa '7cbj Ybcf'hc'Ybfc``]b'h\Y'Wti fgY"

7 ci fgY @YUfb]b[Ci hWca Yg

- %" 8 Yg][bž'XYj Y`cd'UbX'fYdcfhcb'fYgYUfW\'UWfj]h]Yg'UbX'ei Ygh]cbgžg\ck]b['Uk UfYbYgg'cZfYgYUfW\ Yth]Wg'UbX'gY`YWfb['Uddfcdf]UhY'fYgYUfW\ 'a Yth\cXg'fY`UhYX'bc'th\Y]f'fYgYUfW\ 'bcd]Wg/
- &" 7 f]h]WU``mYb[U[Y`k]h\ `W ffYbhgW\c`Ufg\]d`]b`h\Y]f`Z]Y`X`cZgh Xmhc`]bZcfa `fYgYUfW\`UbX`Wcbhf]Vi hY hc`_bck `YX[Y`]b`h\]g``Z]Y`X/UbX
- ' "8Ya cbghfuhY`fYgci fWYZI`bYggž'WfYuhjj]lmiUbX`ZYI]V]`]lmi]b`h\Y`Udd`]WUhjcb`cZ_bck `YX[Y`UbX`g_]``g`hc gc`j Y`dfcV`Ya g`fY`UhYX`hc`fYgYUfW\"

HYUWX]b['GhfUhY[]Yq'

H\]g`Wci fgY`]g`XYg][bYX`hc`ZUW]`]hUhY`X]fYWYX`fYgYUfW\`i g]b[`cb`]bY`fYgci fWYg`UbX`]bX]j]Xi U`#[fci d gi dYfj]g]cb"`Ghi XYbhg`k]``VY`YbWci fU[YX`hc`XYg][b`U`XYg_!VUgYX`dfc`YWih\Uh\Wb`VY`i bXYfhU_Yb`k]h\]b`U g\cfhiha Y`ZfUa Yz`hc`Udd`mih\Y`WcbWYdhg`X]gWi ggYX`]b`GF 5D) %\$%F YgYUfW\`A Yh\cXg"`H\Y`Wci fgY ghi Wi fY`]bWi XYg`XYX]WUhYX`k f]h]b[`h]a Y`hc`Ybgi fY`ghi XYbhg`Wca d`YhY`h\Y]f`fYgYUfW\`dfc`YWig`k]h\]b qhUhYX`h]a YZfUa Yg"

5ggYgga Ybh

7ci fgY'XYg][b'UbX'`YUfb]b['UWh]j]h]Yg'

 $H\[g'Wci\ fgY'\Ug'VYYb'XYg][\ bYX'hc'ZJW]']\[bVX'hc'ZJW]'\]bVX'fYgYUfWX''FYYGYUfWX''FYYGCI fW'g'k\]```VY'Uj U]'UV'Y'cb']\[bY'UbX'\] fci d'UbX#cf']bX]j]Xi U``gi dYfj]g]cb'k\]```HU_Y'd'UW'']b'U'ZJWY'hc'ZJWY'Ybj]fcba YbH''GHi XYbhg'Wb'i bXYfHU_Y [fci d'k cf_'UbX'dYYf'fYj]Yk'h\fci [\ci h'h\Y'Wci fgYž'\ck Yj Yfž'h\Y'Z]bU``YggUmcf'fYgYUfWX'dfc'YWhk\]``VYUb']bX]j]Xi U``d]YWY'cZk cf_'''GHi XYbhg'WcbXi WhU'd]YWY'cZXYg_!VUgYX'fYgYUfWX'cb'U'hcd]WcZh\Y]f W\ccg]b['''H\Y'Wci fgY'[]j Yg'h\Y'ghi XYbhg'h\Y'cddcfhi b]mhc'WcbXi WhfYgYUfWX']b'Ub'UfYU'cZ]bhYfYghik]h\]b$

Uj Uj`UV`Y`hc`gh`XYbhg`Uhih\Y`ghUfhcZh\Y`gYa YghYf`gc`h\Uhih\YmWUb`k cf_hc`gdYWjZjYX`ghUbXUfXg"`

5ggYgga Ybh'& :]bU`F Ydcfh

GhUfh XUhY.

@Yb[h\.(ž\$\$\$ k cfXg

8 YHJ]`g. `Ghi XYbhg`gi Va]h`Ub`]bX]j]Xi U``fYgYUfW\fYdcfhcZ(\$\$\$`k cfXg`cb`h\Y`XYg_!VUgYX`fYgYUfW\h\Ym WcbXi WhYX"`K f]hhYb`ZYYXVUW_`UbX`U`bi a Yf]WJ``[fUXY`k]```VY`[]j Yb`hc`ghi XYbhg`k]h\]b`&`k YY_g`cZXY`]j Yfm' A Uf_]b[`Wf]hYf]U`k]```VY`a UXY`Uj U]`UV`Y`hc`ghi XYbhg`Uhih\Y`ghUfhcZh\Y`gYa YghYf`gc`h\Uhih\Ym\WJb`k cf_`hc gdYW]Z]YX`ghUbXUfXg"``H\]g`]g`h\Y`Z]bU``UggYgga YbhZcf`UhhYbXUbW\`di fdcgYg"

 $\label{thm:continuous} \begin{tabular}{ll} Hi & fb & fb & gYM &$

5 https://bxubwww.Fyei]fya Ybhg

Ghi XYbhgʻUfYʻghfcb[`miYbWci fU[YX'hcʻUhhYbX'U```WUggYgʻUbX'fYj]Yk '`YWhi fY`fYWcfX]b[g"

7ci fgY'GWXYXi `Y

J]Yk 'WUgg'ha YhUV'Y

H]a YtUV`Y

8UhY	Hmd Y	7 cbhYbh
K YY_'%'' '>i bY'!'+'>i bY	GYa]bUf	; YM]b['gHJfhYX'cb'mci f'fYgYUfVX'
		7 ci fgY cj Yfj]Yk 'UbX'UggYgga Ybhg
		K \Uh]g:XYg_!VUgYX:fYgYUfW\3
		; YbYfU``]blfcXi Wf]cb'hc'fYgYUfW\!'8Yj Y`cd]b['U WcbWfdha Ud'UVci hmci f'fYgYUfW\'hcd]W
		EfgVW fei [^^WShWfZ[eeW_ [Sdg`VWbefS`V[Y iZSf[e[ha^hWV[Xb_ [YfZW[ddWeWSdJZbdaT^W_ S`Vg`VWbfS][YSVWe]ŽTSeWVdWeWSdJZbdaWUfz
K YY_"&."%\$">i bY"!"%(>i bY	F YUX]b[Di V`]W <c`]xuml'ei td="" yybfg'6]fh\xum<=""></c`]xuml'ei>
> D1		
		Bc'gYa]bUf'cf']bX]j]XiU``gidYfj]g]cb
K YY_'' .'%+'>i bY'!'&% >i bY	GYa]bUf	FYZ]b]b['mci f'fYgYUfW\'dfcYWh
> U1		6YWca]b['U'k f]hYf'!'GhUfh'k f]h]b['''
		EfgVW fei [^Tq`Y 'Sbfabe adi qff`Y _ SfWqS'ežI W TWY[i qff`Y [e[W UWS`V efab %" _ [gfWe 'SfWd XadS #" _ [TdWS] žFZW i WeWffWVai ` SYS[Xad S`afZWd%" _ [gfWežEa e[_bWTgf [fi ad] e FZ[e [VWS [e fa YWf kag fa i qftWXda_ S` WSdk efSYWaXfZW dWeWSdJZ dWbadf bdaUWeež;fUS``af TWefdWeeW faa _ gUZ Zai [_badfS`f [f [e fa i qftWi qftWS`V i qffW
		GYUfWk]b['Zcf'fYgYUfWk'
		GYWcbXUfmei UbhjhUhjj Y`XUhU`gci fWrg`!`Yl]ghjb[[cj Yfba Ybh'UbX`B; C`XUhU`gYhg`fY"["'Wrbgi gŁ'''
		DYYf'fYj]YkYX''ci fbU'g'!'fUd]X'Yj]XYbW''fYj]Yk g
		; fYm`]hYfUh fY'!'i bdi V`]g\YX'`]hYfUh fY'UbX'fYdcfhg dfcXi WYX'Vm[cjYfbaYbhUbX'B; Cg"'''''

	EfgVW fei [^^WShWfZ[eeWee[a`g`VWdefS`V[`YhSdk[`YeagdJWeaX[`Xad_Sf[a``ffWdSfgdWfZSfUS`

		dfYgYbhJb['mci f'fYgYUfW\"'6cc_YX'hJa Yg'k]h\ Wci fgY'WccfX]bUhcf"
K YY_''&-'>i`m!'& 5i[igh	GYa]bUf	@ b_]b[i d !]bhYfdfYh]b[mci f Z]bX]b[g F Yj]Yk]b['\ck mci f Z]bX]b[g `)b_'hc h\YcfmUbX
		fY`UHYX``]HYFUH fY"
		A U_]b['U'glfcb['Uf[i a Ybh3' < cb]b[']b 'cb 'k \Uh a UhhYfg"
		8 cYgʻmci fʻYj]XYbWYʻgi ddcfhmci fʻUf[i a Ybh3
K YY_'%\$.') '5i [i gh!'- 5i [i gh	GYa]bUf	⇒YbhjZmjb[ˈh\Yˈ]a d`]WUhjcbgʻcZmci fʻfYgYUfW\dfc^YWh
		K \miXcimci fiZjbXjb[gia UthYf3
		K \UhiUfY'h\Y']a d`]WUh]cbg'cZmci f'fYgYUfW\'!'Zcf dc`]WmUbX#cf'dfUWh]WYZ'UXj cWUWmUbX'Z fh\Yf fYgYUfW\3
K YY_'%%''%&'5i [i gh!'%* 5i [i gh	< ca Yk cf_	±xYdYbXYbhfYgYUfW\`UbX#cf`]bX]j]Xi U` gi dYfj]g]cb
		Ghi XYbhgʻk cf_ʻhck UfXgʻgi Va]hh]b[ʻZ]bU``fYgYUfVX fYdcfh'ʻ
		6cc_YX'h]a Yg'k]h\'WtifgY'WtbjYbcf"
		KagdX(`S^UZS`UWfa_WWfS`VV[eUgee kagdbdaWUfTWAdWegT_[ee[a`ž

FYgci fWyg

DfYgWf]VYX F Ygci fWyg

Mci 'k]```VYbYZ]hZfca 'VYWca]b['ZJa]`]Uf'k]h\; cc[`Y'GW\c`Uf'f\hd.#gW\c`Uf'[cc[`Y'Wca Ł'Ug'U'_YmgYUfW\ Yb[]bY'Zcf'UWXYa]Wdi V`]WUh]cbg'UbX'fYdcfhg"'Mci 'WUb'gYhi d'h\Y'dfYZYfYbWYg'hc'`]b_'hc'h\Y'I BGK @]VfUfmYj Yb'k \Yb'mci 'UfY'bch'cb'WJa di g"

; chc; $cc[Y'GWkcUf2gYhh]b[g2]VfUfmi]b_gžUbXYbhYf'fli b]j Yfg]hmicZBYk 'Gci h\ 'K UYgfi]b'h\Y'Vcl 'Zcf'h\Y' @VfUfm''"$

FYWca a YbXYX FYgci fWyg

6Y'W.Yfz'K "'f&\$\$- L'I dff' Y kagd agd S^Sdf[UW[' #\$ i WW] ez'@cg'5b['Yg. 'G5; 9"

<U``žF"'f&\$\$, Ł'3bb'fWW EaU[S^DWeWSdJZł GmXbYm'A UWA]``Ub"''

7ci fgY 9j U'i Uhjcb UbX 8Yj Y'cda Ybh

Gh XYbhYj U`i Uhjj Y`ZYYXVUW_`]g`[Uh\YfYX`dYf]cX]WU``mi g]b[žUa cb[`ch\Yf`a YUbgžI BGK fig`a m9I dYf]YbWYdfcWYgg"' ≠bZcfa U``ZYYXVUW_`UbX`WUgg![YbYfUhYX`ZYYXVUW_`UfY`U`gc`]a dcffUbh'`Gh XYbhZYYXVUW_`]g`hU_Yb gYf]ci g`mUbX`Wcbhjbi U``]a dfcj Ya Ybhg`UfY`a UXY`hc`h\Y`Wci fgY`VUgYX`]b`dUfhcb`gi W\`ZYYXVUW_" G][b]Z]WUbhWXUb[Yg`hc`h\Y`Wci fgY`k]```VY`Wca a i b]WUhYX`hc`gi VgYei YbhWc\cfrg`cZgh XYbhg`hU_]b['h\Y Wci faY"`````

5 WUXYa | W < cbYghmUbX D`U[| Uf]ga

D`U[]Uf]ga `]g`i g]b['h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX`dfYgYbh]b['h\Ya `Ug`mci f`ck b"'=hWJb'hU_Y`a Ubm Zcfa gžZfca `XY`]VYfUhY`W\YUh]b['hc`UWW]XYbhU`mWcdm]b['Zfca `U`gci fWY'k]h\ci hUW_bck `YX[Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga]bhc'h\Y'Zc``ck]b[WUhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl h'cf']XYU'k]h\ci h'UW_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf_g"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k\YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'

♣Uddfcdf]UhY 'dUfUd\fUg]b[...7\Ub[]b[...7\Ub[]b[...7\Ub[]b[...7\Ub[]b[...7\Ub[]b]b[...7\Ub[]b]b[...7\Ub[]b]b[...7\Ub[]b]b[...7\Ub[]b]b[...7\Ub[]b]b[...7\Ub[]b]b[...7\Ub[]b]b[...7\Ub[]b]b[...7\Ub[]b]b[...7\Ub[]b]b[...7\Ub[]b]b[...7\Ub]b]b[...7\Ub]b[...7\Ub]b[...7\Ub]b]b[...7\Ub]b[...7\Ub]b[...7\Ub]b[...7\Ub]b[...7\Ub]b[...7\Ub]b]b[...7\Ub]b[...7\Ub]b[...7\Ub]b[...7\Ub]b]b[...7\Ub]b[...7\Ub]b[...7\Ub]b[...7\Ub]b[...7\Ub]b[...7\Ub]b]b[...7\Ub]b[...7\Ub]b[...7\Ub]b[...7\Ub]b]b[...7\Ub]b[...7\Ub]b[...7\Ub]b]b[...7\Ub]b[...7\Ub]b[...7\Ub]b[...7\Ub]b[...7\Ub]b]b[...7\Ub]b[...7\Ub]b[...7\Ub]b[...7\Ub]b]b[...7\Ub]b[...7\Ub]b[...7\Ub]b]b[...7\Ub]b[...7\Ub]b[...7\Ub]b[...7\Ub]b]b[...7\Ub]b[...7\Ub]b[...7\Ub]b[...7\Ub]b[...7\Ub]b[...7\Ub]b[...7\Ub]b]b[...7\Ub]b[...7\Ub]b[...7\Ub]b[...7\Ub]b[...7\Ub]b[...7\Ub]b[...7\Ub]b]b[...7\Ub]b[...7\Ub]b[...7\Ub]b[...7\Ub]b[...7\Ub]b[...7\Ub]b]b[...7\Ub

7 c``i g]cb.'k cf_jb['k]h\ch\Yfg'Vi hdUgg]b['cZZh\Y'k cf_'Ug'U'dYfgcb'g']bX]j]Xi U``k cf_"'7 c``i g]cb'U`gc]bWi XYg'dfcj]X]b['mci f'k cf_'hc'Ubch\Yf'ghi XYbhVYZcfY'h\Y'Xi Y'XUhYžcf'Zcf'h\Y'di fdcgY'cZh\Ya d`U[]Uf]g]b['UhiUbmh]a Yž'dUm]b['Ubch\Yf'dYfgcb'hc'dYfZcfa 'Ub'UWUXYa]WhUg_ž'ghYU']b['cf'UWei]f]b[Ubch\Yf'dYfgcb'g'UWUXYa]Wk cf_'UbX'Wcdm]b[']hz'cZYf]b['hc'Wca d'YhY'Ubch\Yf'dYfgcb'g'k cf_'cf'gYY_]b[dUma YbhZcf'Wca d'Yh]b['UWUXYa]Wk cf_"

±bUddfcdf]UhY`W]hUh]cb.`7]h]b[`gcifWrg`k\]W\`\UjY`bchVYYb`fYUXžk]h\cihUW_bck`YX[]b[`h\Y`´gYWcbXUfm´gcifWrZfca`k\]W\`_bck`YX[Y`cZh\Ya`\Ug`VYYb`cVhU]bYX"

8 i d`]\WUh]cb`ffgY`Zd`U[]\Uf]ga "Ł gi Va]\h]b[`mci f`ck b`k cf_ž]b`k \c`Y`cf`]b`dUfhžk \YfY`]h\Ug`dfYj]ci g`m VYYb`dfYdUfYX`cf`gi Va]\hYX`Zcf`Ubch\Yf`UggYgga Ybhcf\Wci fgY`UhI BGK `cf`Ubch\Yf`i b]j Yfg]\m'

7 cffYWnfYZYfYbW]b[dfUWf]WYg.

- DUfUd\fug]b[ž'gi a a Uf]g]b[ž'YggUmk f]h]b['UbX'h]a Y'a UbU[Ya Ybh
- 5ddfcdf]UhY'i gY'cZUbX'Uhtf]Vi h]cb'Zcf'U'fUb[Y'cZa UhYf]U`g']bWi X]b['hYl hž']a U[Ygž'Zcfa i `UY'UbX WbWYdhg"

±0X]j]Xi U``Ugg]ghUbWr`]g`Uj U]`UV`Y`cb`fYei YghZfca `H\Y`@YUfb]b[`7 YbhfY`f\hd.#k k k "`Wi bgk "YXi "Ui #L" Ghi XYbhg`UfY`U`gc`fYa]bXYX`h\UhWUfYZ ``ha Ya UbU[Ya Ybh]g`Ub`]a dcfhUbhdUfhcZghi XmUbX`cbY`cZh\Y]XYbh]Z]YX`WUi gYg`cZd`U[]Uf]ga `]g`dccf`h]a Y`a UbU[Ya Ybh'`Ghi XYbhg`g\ci `X`U``ck `gi ZZ]W]Ybh'h]a Y`Zcf fYgYUfW\žXfUZn]b[`UbX`dfcdYf`fYZYfYbW]b[`cZgci fW*g`]b`dfYdUf]b[`U```UggYgga Ybh']hYa g"

IBGK "@]VfUfmU'gc'\Ug'h\Y'9@&9'hcc`'Uj U]`UV`Y'hc'Ugg]ghimci ik]h\'mci f'ghi Xm'Uhi BGK "'9@&9']g XYg][bYX'hc']bhfcXi WY'bYk 'ghi XYbhg'hc'ghi Xm]b['Uhil BGK 'Vi h']h\Wb'U'gc'VY'U'[fYUhifYZfYg\Yf'Xi f]b['mci f ghi Xm'

7 ca d'Yhjb['h\Y'9@&9'hi hcf]U`'UbX'ei]n'k]``'YbUV'Y'mci 'hc.

- UbU`mgY`hcd]\Mgž'd`Ub`fYgdcbgYg`UbX`cf[Ub]gY`fYgYUf\X\`Zcf`U\WJXYa]\Wkf]hjb[`UbX`ch\YfUggYggaYbhiHJg_g
- YZZYWIJ Y`mUbX`YZZWYbh`mZbX`Uddfcdf]UhY`]bZcfa Uh]cb`gci fWg`UbX`Yj U`i UhY`fY`Yj UbWY`hc`mci f bYYXg
- i gY`UbX`a UbU[Y`]bZcfa Uh]cb`YZZYWf]jY`mihc`UVWca d`]g\`U`gdYW]Z]Wdi fdcgY



5 WUX Ya] W = D Zcfa Uh] cb ·

: cf`YggYbhJU``gh XYbh]bZcfa UhJcb`fY`UhJb['hc.

fYei YghgʻZcfʻYl hYbg]cb/ `UhYʻgi Va]gg]cbgʻ[i]XY`]bYg/ fYj]Yk ʻcZa Uf_g/ I BGK ʻ< YUʻh\ 'UbXʻGUZYhmdc`]WJYg/